**Radical Plastic Surgery**

#1) In order to pay for radical plastic surgery, George decides he needs a business plan to pay for said surgery. George's entrepreneur spirit has led him to building toys and selling them online. George can sell 12 remote controlled double helix helicopters per day at a price of $150. He estimates that for each $3 price reduction he can sell two more helicopters per day. If each helicopter costs him $120 to make and fixed costs are $10, what price should he charge to maximize his profit?

$x =$ # of price reductions

$q(x) = 12 + 2x$

$p(x) = 150 - 3x$

$R(x) = p(x)\,q(x)$
$= (150 - 3x)(12 + 2x)$
$= 1800 - 36x + 300x - 6x^2$
$R(x) = 1800 + 264x - 6x^2$

$C(x) = (\$\text{unit})\,q(x) + f.c.$
$C(x) = 120(12 + 2x) + 10$
$C(x) = 1450 + 240x$

$P(x) = R(x) - C(x)$
$= (1800 + 264x - 6x^2) - (1450 + 240x)$
$P(x) = -6x^2 + 24x + 350$

$P'(x) = -12x + 24$
$0 = -12x + 24$
$12x = 24$
$x = 2$

$P''(x) = -12$
$P''(2) = $ neg, CCD, MAX

**What price should be charged per item?**

$p(2) = 150 - 3(2)$
$= 150 - 6$
$p(2) = \$144$

**How many helicopters will he sell at this price?**

$q(2) = 12 + 2(2)$
$= 12 + 4$
$q(2) = 16$ helicopters

**What is his maximum profit?**

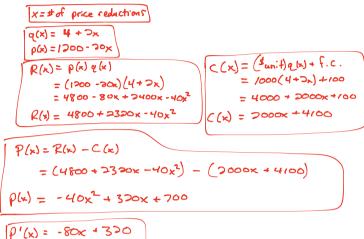$P(2) = -6(2)^2 + 24(2) + 350$
$= -6(4) + 48 + 350$
$= -24 + 398$
$P(2) = \$374$

**Sentence answer:**

George should make 2 price reductions at $3 each. He will sell 16 helicopters at $144 each with max profit of $374 per day.

**Turbo Lawn Mowers**

#2) George can sell four turbo charged lawn mowers per day at a price of $1,200. He estimates that for each $20 price reduction, he can sell two more mowers per day. If each car costs him $1,000 and his fixed costs are $100, what price should he charge to maximize his profit?

$x =$ # of price reductions

$q(x) = 4 + 2x$

$p(x) = 1200 - 20x$

$R(x) = p(x)\,q(x)$
$= (1200 - 20x)(4 + 2x)$
$= 4800 - 80x + 2400x - 40x^2$
$R(x) = 4800 + 2320x - 40x^2$

$C(x) = (\$\text{unit})\,q(x) + f.c.$
$= 1000(4 + 2x) + 100$
$= 4000 + 2000x + 100$
$C(x) = 2000x + 4100$

$P(x) = R(x) - C(x)$
$= (4800 + 2320x - 40x^2) - (2000x + 4100)$
$P(x) = -40x^2 + 320x + 700$

$P'(x) = -80x + 320$
$0 = -80x + 320$
$80x = 320$
$x = 4$

$P''(x) = -80$
$P''(4) = $ neg, CCD, MAX

**What price should be charged per item?**

$p(4) = 1200 - 20(4)$
$= 1200 - 80$
$p(4) = \$1120$

**How many mowers will he sell at this price?**

$q(4) = 4 + 2(4)$
$= 4 + 8$
$q(4) = 12$ mowers

**What is his maximum profit?**

$P(4) = -40(4)^2 + 320(4) + 700$
$= -40(16) + 1280 + 700$
$= -640 + 1980$
$P(4) = \$1340$

**Sentence answer:**

George should make 4 price reductions at $20 each. He will sell 20 mowers at $1120 each with max profit of $1340 per day

**Scalping Cavaliers**

#3) George finds that if he sells a scalped Cavs ticket at $200, he will sell 300 tickets per day. He estimates that each $10 price reduction will result in 30 more tickets sold per day.

Find the ticket price to maximize the revenue.

$x$ = # of price reductions

$p(x) = \$200 - \frac{\$}{10}x$

$q(x) = 300 + 30x$

$R(x) = p(x)\,q(x)$

$\quad = (200 - 10x)(300 + 30x)$

$\quad = 60{,}000 + 6000x - 3000x - 300x^2$

$R(x) = 60{,}000 + 3000x - 300x^2$

$R'(x) = 3000 - 600x$

$0 = 3000 - 600x$

$600x = 3000$

$x = 5$

$R''(x) = -600$

$R''(5) = $ neg, CCD, MAX

What price should be charged per item?

$p(5) = \$200 - \frac{\$}{10}(5)$

$\quad = 200 - 50$

$p(5) = \$150$

How many tickets would be sold at the maximum revenue?

$q(5) = 300 + 30(5)$

$\quad = 300 + 150$

$q(5) = 450$ tickets

What is the maximum revenue?

$R(5) = 60{,}000 + 3000(5) - 300(5)^2$

$\quad = 60{,}000 + 15000 - 300(25)$

$\quad = 75{,}000 - 7500$

$R(5) = \$67{,}500$

Sentence answer:

George should make 5 price reductions at $10 each. He will sell 450 tickets at $150 each with max revenue of $67,500 per day.

**Barrels of Monkey**

#4) George produces barrels of monkey. He can sell 1 million barrels of monkey a day at a price of $25 per barrel. If each $1 price increase results in a sales decrease of 50,000 barrels per day, what price will maximize the George's revenue?

$x$ = number of price increases

$p(x) = \$25 + x$

$q(x) = 1{,}000{,}000 - 50{,}000x$

$R(x) = p(x)\,q(x)$

$\quad = (25 + x)(1{,}000{,}000 - 50{,}000x)$

$\quad = 25{,}000{,}000 - 1{,}250{,}000x + 1{,}000{,}000x - 50{,}000x^2$

$R(x) = 25{,}000{,}000 - 250{,}000x - 50{,}000x^2$

$R'(x) = -250{,}000 - 100{,}000x$

$0 = -250{,}000 - 100{,}000x$

$100{,}000x = -250{,}000$

$x = -2.5$

$R''(x) = -100{,}000$

$R''(-2.5) = $ neg, CCD, MAX

What price should be charged per item?

$p(-2.5) = 25 + (-2.5)$

$p(-2.5) = \$22.50$ per barrel

How many barrels will it sell at this price?

$q(-2.5) = 1{,}000{,}000 - 50{,}000(-2.5)$

$\quad = 1{,}000{,}000 + 125000$

$q(-2.5) = 1{,}125{,}000$ barrels of monkey

What is the maximum revenue?

$R(-2.5) = p(-2.5)\,q(-2.5)$

$\quad = (\$22.50)(1{,}125{,}000)$

$\quad = \$25{,}312{,}500$

Sentence answer:

George should make 2.5 price increases at $1 each. He will sell 1,125,000 barrels at $22.50 a barrel with max revenue of $25,312,500 per day.

**Renting Furniture**

#5) Since his parents work 48 hour shifts on the weekend, George decides to rent out his parents furniture while they are at work. He finds that he can rent 60 pieces of furniture if he charges $80 for a weekend. He estimates that for each $5 price increase it will rent three fewer pieces of furniture. What price should he charge to maximize its revenue?

$x = $ # of price increases

$p(x) = 80 + 5x$
$q(x) = 60 - 3x$

$R(x) = p(x) q(x)$

$\quad = (80 + 5x)(60 - 3x)$

$\quad = 4800 + 300x - 240x - 15x^2$

$R(x) = 4800 + 60x - 15x^2$

$R'(x) = 60 - 30x$
$0 = 60 - 30x$
$30x = 60$
$x = 2$

$R''(x) = -30$
$R''(2) = $ neg, CC DN, MAX

**What price should be charged per item?**

$p(2) = 80 + 5(2)$

$\quad = 80 + 10$

$p(2) = \$90$

**How many cars will it rent at this price?**

$q(2) = 60 - 3(2)$

$\quad = 60 - 6$

$q(2) = 54$ cars

**What is the maximum revenue?**

$R(2) = p(2) \cdot q(2)$

$\quad = (90)(54)$

$R(2) = \$4860$

Sentence answer:

George should make 2 price increases at $5 each. He will rent 54 pieces of furniture at $90 each with max revenue of $4860 per weekend.

**Pollen**

#6) Testing out the pollen market, George planted 20 flowers with each flower yielding 90 bee handfuls of pollen. He estimates that for each additional flower that he plants, the yield of each flower will decrease by 3 bee handfuls. How many flowers should he plant to maximize his pollen farm?

$x = $ # of add'l flowers planted

$f(x) = $ # of flowers $= 20 + x$
$p(x) = $ bee handfuls of pollen $= 90 - 3x$

$fp(x) = (20 + x)(90 - 3x)$

$\quad = 1800 + 90x - 60x - 3x^2$

$fp(x) = 1800 + 30x - 3x^2$

$fp'(x) = 30 - 6x$

$0 = 30 - 6x$

$6x = 30$

$x = 5$

$fp''(x) = -6$

$fp''(x) = $ neg, CCD, MAX

**How many flowers should he plant?**

$f(5) = 20 + (5)$

$f(5) = 25$ flowers

**What is the yield per flower?**

$p(5) = 90 - 3(5)$

$\quad = 90 - 15$

$p(5) = 75$ bee handfuls

**What is his maximum pollen production?**

$fp(5) = f(5) \, p(5)$

$\quad = (25)(75)$

$fp(5) = 1875$ bee handfuls

Sentence answer:

George should plant 5 fewer flowers. This will give him 25 flowers with each flower producing 75 bee handfuls of pollen. His pollen farm will max with 1875 bee handfuls of pollen.

# Advanced Techniques
## 6.1A – Maximizing With a Reduction

**Ant Farm**

#7) George decides to produce and sell ant farms. He sets out 40 peach pits resulting in each peach pit attracting 60 ants. He estimates that for each additional peach pit that he sets out, the attractiveness of each pit will decrease by 2 ants. How many peach pits should he set out to maximize his ant harvest?

$x = \#$ of Add'l peach pits

$P(x) = 40 + x$  (peach pits)

$A(x) = 60 - 2x$  (Ants per pit)

$PA(x) = (40+x)(60-2x)$

$= 2400 - 80x + 60x - 2x^2$

$PA(x) = 2400 - 20x - 2x^2$

$PA'(x) = -20 - 4x$

$0 = -20 - 4x$

$4x = -20$

$x = -5$

$PA''(x) = -4$

$PA''(-5) = $ neg. CCD. MAX

How many peach pits should he set out?

$P(-5) = 40 + (-5)$

$P(-5) = 35$ pits

What is the yield of ants per peach pit?

$A(-5) = 60 - 2(-5) = 60 + 10$

$A(-5) = 70$ Ants

What is the total ant harvest?

$PA(-5) = P(-5)A(-5)$

$= (35)(70)$

$PA(-5) = 2450$ Ants

Sentence answer:

To max ant harvest to 2450 ants, George should set out 5 fewer peach pits. He should set out 35 pits, with each pit attracting 70 ants.

**Neighbors' Cars**

#8) Using Craigslist George can sell 12 of his neighbors' cars per week at a price of $2000 each. He estimates that for each $400 price decrease will result in 3 more sales per week. If it cost him $1200 to hire a thief to steal each car, what price would he charge to maximize his profit?

$x = \#$ of price decreases

$p(x) = 2000 - 400x$

$q(x) = 12 + 3x$

$R(x) = p(x)q(x)$

$= (2000 - 400x)(12 + 3x)$

$= 24000 - 4800x + 6000x - 1200x^2$

$R(x) = 24,000 + 1200x - 1200x^2$

$C(x) = (\text{unit})q(x) + f.c.$

$= 1200(12+3x) + 0$

$C(x) = 14400 + 3600x$

$P(x) = R(x) - C(x)$

$= (24,000 + 1200x - 1200x^2) - (14400 + 3600x)$

$P(x) = 9600 - 2400x - 1200x^2$

$P'(x) = -2400 - 2400x$

$0 = -2400 - 2400x$

$2400x = -2400$

$x = -1$

$P''(x) = -1200$

$P''(-1) = $ neg. CD, MAX

What price would he charge to maximize his profit?

$P(-1) = 2000 - 400(-1)$

$= 2000 + 400$

$P(-1) = \$2400$

How many will he sell at this price?

$q(-1) = 12 + 3(-1)$

$= 12 - 3$

$q(-1) = 9$ cars

What is his maximum profit?

$P(-1) = 9600 - 2400(-1) - 1200(-1)^2$

$= 9600 + 2400 - 1200$

$= 9600 + 1200$

$P(-1) = \$10,800$

Sentence answer:

George should make 1 price increase at $400 each. He will sell 9 cars at $2400 each with max profit of $10,800 per week.