Advanced Techniques
6.1 – Maximizing With a Reduction

When dealing with price reductions, x will be used for the number of price reductions. Quantity will be defined by the context of the problem.

$x = number\ of\ price\ reductions$

$p(x) = (original\ price) - (\$\ per\ reduction)x$

$q(x) = (original\ qty) + (add'n\ sold)x$

$R(x) = p(x) \cdot q(x)$

$C(x) = (Unit\ cost)q(x) + (fixed\ cost)$

$P(x) = R(x) - C(x)$

A computer manufacturer can sell 1500 personal computers per month at a price of \$3000 each. The manufacturer estimates that for each \$200 price reduction he will sell 300 more each month. If *x* stands for *the number of \$200 price reductions*, express the price *p* and the quantity *q* as functions of *x*.

x =

Price of each Computer: p(x) =

Total Quantity Sold: q(x) =

Ex A: Maximizing a Company's Profit

A store can sell 20 bicycles per week at a price of \$400 each. The manager estimates that for each \$10 price reduction she can sell two more bicycles per week. The bicycles cost the store \$200 each.

x = # of \$10 price reductions

$p(x) = \$400 - \$10x$

$q(x) = 20 + 2x$

$R(x) = p(x)\, q(x)$

$= (400 - 10x)(20 + 2x)$

$= 8000 - 200x + 800x - 20x^2$

$R(x) = 8000 + 600x - 20x^2$

$C(x) = (\text{Unit \$})\, q(x) + \text{f.c.}$

$= \$200(20 + 2x)$

$C(x) = 4000 + 400x$

$P(x) = R(x) - C(x)$

$= (8000 + 600x - 20x^2) - (4000 + 400x)$

$P(x) = -20x^2 + 200x + 4000$

$P'(x) = -40x + 200$

$0 = -40x + 200$

$40x = 200$

$x = 5$

$P''(x) = -40$

$P''(5) = \text{neg, CCD, MAX}$

Find the price of the bicycles that maximize profit.

$p(5) = 400 - 10(5)$

$= 400 - 50$

$= \$350$

Find the quantity of the bicycles that maximize profit.

$q(5) = 20 + 2(5)$

$= 20 + 10$

$q(5) = 30$

Also find the maximum profit.

$P(10) = -20(5)^2 + 200(5) + 4000$

$= -20(25) + 1000 + 4000$

$= -500 + 5000$

$P(10) = 5400$

Sentence Answer

If the store makes 5 price reductions of \$10 each, it will sell 30 bikes at \$350 each, giving a max profit of \$5400

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Ex B: Maximizing Harvest Size

An orange grower finds that if he plants 80 orange trees per acre, each tree will yield 60 bushels of oranges. He estimates that for each additional tree that he plants per acre, the yield of each tree will decrease by 2 bushels. How many trees should he plant per acre to maximize his harvest?

$x = \#$ of add'n trees planted

$T(x) = 80 + x$

$Y(x) = 60 - 2x$

$TY(x) = (80+x)(60-2x)$

$= 4800 + 60x - 160x - 2x^2$

$TY(x) = 4800 - 100x - 2x^2$

$TY'(x) = -100 - 4x$

$0 = -100 - 4x$

$4x = -100$

$x = -25$

$TY''(x) = -4$

$TY''(-25) =$ neg, CCD, MAX

$T(-25) = 80 + (-25)$

$T(-25) = 55$ trees/acre

$Y(-25) = 60 - 2(-25)$

$= 60 + 50$

$Y(-25) = 110$ buslels per tree

$TY(-25) = 4800 - 100(-25) - 2(-25)^2$

$= 4800 + 2500 \quad -2(625)$

$= 4800 + 2500 \quad -1250$

$= 7300 \quad -1250$

$TY(-25) = 6050$ bushels per acre

Pro Tips

#1) Define x

Define tree per acre function T(x)

Define yield per tree function Y(x)

Derive total yield per acre function TY(x)

#2) To maximize the yield, set the $TY'(x) = 0$ and solve.

Make sure it is a max by using the second derivative test.

#3) Use x to answer the question asked.

Sentence answer

He should plant 55 trees per acre which will yield 110 bushels per tree, giving a max yield of 3800 bushels per acre.