Advanced Techniques
6.2A – More Maximizing Applications

The relationship between the total sales in millions of dollars, S, and the tax rate, r, is given by each formula. (Write your answer from part a. and b. into one sentence.

a. Find the tax rate that maximizes the government revenue from sales tax.
b. Find the maximum revenue.

#1) $S(t) = 5 - 9\sqrt[3]{t}$

$GR(t) = S(t) \cdot t$

$= (5 - 9t^{1/3})\, t^{3/3}$

$GR(t) = 5t - 9t^{4/3}$

$GR'(t) = 5 - 12t^{1/3}$

$0 = 5 - 12\sqrt[3]{t}$

$12\sqrt[3]{t} = 5$

$\sqrt[3]{t} = \frac{5}{12}$

$t = \left(\frac{5}{12}\right)^3$

$t = \frac{125}{1728}$

$t \approx .072$

$t = 7.2\%$

$GR''(t) = -4t^{-2/3}$

$GR''(.072) = \text{neg, CCD, MAX}$

$GR(.072) = 5(.072) - 9(.072)^{4/3}$

$\approx .36 - 9(.072)^{4/3}$

$\approx .090421 \text{ million}$

$GR(.072) \approx \$90{,}421$

Sentence answer:

The gov't should set the tax rate at 7.2%. This will max their revenue at about \$90,421.

#2) $S(t) = 10 - 21\sqrt[3]{t}$

$GR(t) = S(t) \cdot t$

$= (10 - 21t^{1/3})\, t^{3/3}$

$GR(t) = 10t - 21t^{4/3}$

$GR'(t) = 10 - 28t^{1/3}$

$0 = 10 - 28\sqrt[3]{t}$

$28\sqrt[3]{t} = 10$

$\sqrt[3]{t} = \frac{10}{28}$

$t = \left(\frac{5}{14}\right)^3$

$t = \frac{125}{2744}$

$t \approx .046$

$t = 4.6\%$

$GR''(t) = -\frac{28}{3}t^{-2/3}$

$GR''(.046) = \text{neg, CCD, MAX}$

$GR(.046) \approx 10(.046) - 21(.046)^{4/3}$

$\approx .46 - 21(.046)^{4/3}$

$\approx .113878 \text{ million}$

$GR(.046) \approx \$113{,}878$

Sentence answer:

The gov't should set the tax rate at 4.6%. This will max their revenue at about \$113,878.

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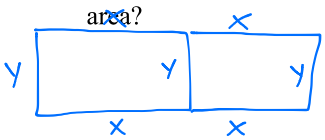
Marshmallow Fence

#3) George has 1200 marshmallows to build a fence around his Hello Kitty collection. He wishes to build two identical rectangular enclosures that share a common side. What should the dimensions of each marshmallow enclosure be in order to maximize the area?



x = length of noncommon side
y = length of common side
A = Area
P = perimeter

$$A = 2xy$$
$$A = 2x\left(400 - \tfrac{4}{3}x\right)$$
$$A = 800x - \tfrac{8}{3}x^2$$
$$A' = 800 - \tfrac{16}{3}x$$
$$0 = 800 - \tfrac{16}{3}x$$
$$\tfrac{16}{3}x = 800$$
$$16x = 2400$$
$$x = 150$$

$$A'' = -\tfrac{16}{3}$$
$A''(150)$ = neg, CCD, MAX

$$P = 4x + 3y$$
$$1200 = 4x + 3y$$
$$1200 - 4x = 3y$$
$$400 - \tfrac{4}{3}x = y$$

$$400 - \tfrac{4}{3}(150) = y$$
$$400 - 200 = y$$
$$200 = y$$

Dimensions are 150 × 200 marshmellows

What is the maximum area?

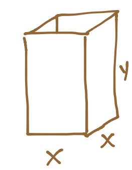
$$A = 2xy$$
$$A_{(150,200)} = 2(150)(200)$$
$$A_{(150,200)} = 60{,}000$$

Sentence answer:

To max area at 60,000 square marshmallows, each enclosure should be 150 by 200, where the common side is 200.

Bear Trap

#4) George wants to make a bear trap to catch squirrels. According to Reddit, he needs to dig a rectangular hole with square base with an exact volume of 4 cubic feet. Find the dimensions of the hole that can be made with the smallest dirt surface area. (Hint, the dirt surface area is the bottom and the four lateral sides of the hole. There is no dirt top of the hole. /hint)



x = length of square base
y = height
V = Volume
SA = Surface Area

$$SA = A_{Front} + A_{BACK} + A_{left} + A_{Right} + A_{Bottom}$$
$$= xy + xy + xy + xy + x \cdot x$$
$$SA = 4xy + x^2$$
$$SA = 4x\left(\tfrac{4}{x^2}\right) + x^2$$
$$SA(x) = 16x^{-1} + x^2$$

$$V = A_{base} \cdot \text{height}$$
$$4 = x^2 y$$
$$\tfrac{4}{x^2} = y$$

$$SA'(x) = -16x^{-2} + 2x$$
$$0 = \tfrac{-16}{x^2} + 2x$$
$$0 = -16 + 2x^3$$
$$16 = 2x^3$$
$$8 = x^3$$
$$2 = x$$

$$\tfrac{4}{(2)^2} = y$$
$$\tfrac{4}{4} = y$$
$$1 = y$$

$$SA''(x) = 32x^{-3} + 2$$
$SA''(2)$ = pos, CCU, MIN

What is smallest amount of material?

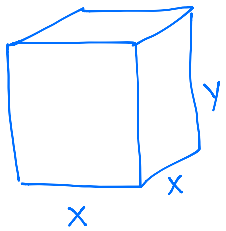
$$SA_{(2,1)} = 4(2)(1) + (2)^2$$
$$= 8 + 4$$
$$SA_{(2,1)} = 12 \text{ ft}^2$$

Sentence answer:

To min surface area of hole to 12 ft^2, the square base should be 2 feet and have a height of 1 foot.

Bob – the Sponge

#5) George needs to make an open-top aquarium with square base. The volume must be precisely 108 cubic feet to accommodate his pet sponge, Bob. Find the dimensions of the aquarium that can be made with the smallest amount of material.



x = length of square base
y = height
m = material (SA)
V = Volume

$$M(x,y) = A_{Bottom} + A_{left} + A_{Right} + A_{Front} + A_{Back}$$
$$= x^2 + xy + xy + xy + xy$$
$$M(x,y) = x^2 + 4xy$$
$$M(x) = x^2 + 4x\left(\frac{108}{x^2}\right)$$
$$M(x) = x^2 + 432x^{-1}$$

$$V = A_{base} \cdot \text{height}$$
$$108 = x^2 y$$
$$\frac{108}{x^2} = y$$

$$M'(x) = 2x - 432x^{-2}$$
$$0 = 2x - \frac{432}{x^2}$$
$$0 = 2x^3 - 432$$
$$432 = 2x^3$$
$$216 = x^3$$
$$6 = x$$

$$\frac{108}{(6)^2} = y$$
$$\frac{108}{36} = y$$
$$3 = y$$

$$M''(x) = 2 + 864x^{-3}$$
$$M''(6) = \text{pos, CCU, min}$$

The base is 6 feet by 6 feet. The height is 3 feet.

What is smallest amount of material?

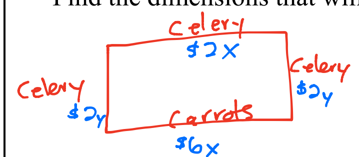
$$M(6,3) = (6)^2 + 4(6)(3)$$
$$= 36 + 72$$
$$M(6,3) = 108$$

Sentence answer:
The square base should be 6 feet and the height of 3 feet. This will min the material used at 108 ft².

Bunny Farm

#6) Wanting to start a bunny rabbit farm, George wants to build the rectangular enclosure using celery and carrot sticks as fencing with the enclosure being exactly 800 square feet. The fencing closest to his house is to be made of carrot sticks and costs $6 per foot; however, the other three sides consists of celery sticks costing only $2 per foot.

Find the dimensions that will minimize the cost.



x = length of carrot sides
y = length of celery sides
C = Cost
A = Area

$$C(x,y) = 2x + 6x + 2y + 2y$$
$$C(x,y) = 8x + 4y$$
$$C(x) = 8x + 4\left(\frac{800}{x}\right)$$
$$C(x) = 8x + 3200x^{-1}$$

$$A = xy$$
$$800 = xy$$
$$\frac{800}{x} = y$$

$$C'(x) = 8 - 3200x^{-2}$$
$$0 = 8 - \frac{3200}{x^2}$$
$$0 = 8x^2 - 3200$$
$$3200 = 8x^2$$
$$400 = x^2$$
$$\pm 20 = x$$

$$\frac{800}{(20)} \approx y$$
$$40 \approx y$$

The front & back should be 20 feet and the left & right should be 40 feet long.

$$C''(x) = 6400x^{-3}$$
$$C''(-20) = \text{neg, CCD, max}$$
$$C''(20) = \text{pos, CCU, min}$$

What is the minimum cost?

$$C(20) \approx 8(20) + \frac{3200}{20}$$
$$\approx 160 + 160$$
$$C(20) \approx \$320$$

Sentence answer:
The front and back should be 20 ft long and the other sides 40 ft, min the cost to $320.

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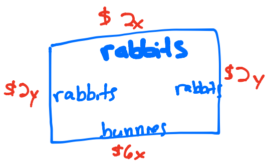
Carrots & Celery

#7) George wants to build a carrot and celery garden enclosed by a fence made of bunnies and rabbits. The garden is to be 5000 square feet. If the fence along the front is made from bunnies and costs \$6 per foot but on the other three sides is made of rabbits and costs only \$2 per foot, find the dimensions that will minimize the cost.



x = length of front & back
y = length of other sides
C = Cost
A = Area

$$C(x,y) = 2x + 6x + 2y + 2y$$
$$C(x,y) = 8x + 4y$$
$$C(x) = 8x + 4\left(\frac{5000}{x}\right)$$
$$C(x) = 8x + 20{,}000x^{-1}$$

$$A = xy$$
$$5000 = xy$$
$$\frac{5000}{x} = y$$

$$C'(x) = 8 - 20{,}000x^{-2}$$
$$0 = 8 - 20{,}000x^{-2}$$
$$0 = 8x^2 - 20{,}000$$
$$20{,}000 = 8x^2$$
$$2500 = x^2$$
$$\pm 50 = x$$

$$\frac{5000}{50} = y$$
$$100 = y$$

The front & back should be 50 feet and the other sides are 100 feet.

$$C''(x) = 40{,}000x^{-3}$$
$$C''(-50) = \text{neg, CCD, MAX}$$
$$C''(50) = \text{pos, CCU, MIN}$$

What is the minimum cost?

$$C(50) = 8(50) + 4(100)$$
$$= 400 + 400$$
$$C(50) = \$800$$

Sentence answer:

The front and back should be 50 ft long and the other sides 100 ft, min the cost to \$800.

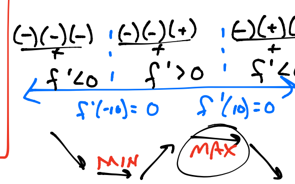
Cookies for Friends

#8) George estimates that by giving away cookies for x days, he will gain $2x$ friends, but his cookie expenses will be $5x^2 + 500$ dollars. He wants to give away cookies the number of days that maximizes the number of friends per dollar, $f(x) = \frac{2x}{5x^2+500}$ dollars. For how many days should he give away cookies?

$f(x)$ = # of friends per \$
x = days

$$f(x) = \frac{2x}{5x^2+500}$$
$$f'(x) = \frac{(2x)'(5x^2+500) - (2x)(5x^2+500)'}{(5x^2+500)^2}$$
$$= \frac{2(5x^2+500) - 2x(10x)}{(5x^2+500)^2}$$
$$= \frac{10x^2 + 1000 - 20x^2}{(5x^2+500)^2}$$
$$0 = \frac{-10x^2 + 1000}{(5x^2+500)^2}$$
$$0 = \frac{-10(x^2 - 100)}{(5x^2+500)^2}$$
$$0 = \frac{-10(x-10)(x+10)}{(5x^2+500)^2}$$

$$f' = \frac{-10(x-10)(x+10)}{(5x^2+500)^2}$$



ZON

$$-10(x-10)(x+10) = 0$$
$$-10 \neq 0 \qquad x - 10 = 0 \qquad x + 10 = 0$$
$$x = 10 \qquad x = -10$$
$$\text{CV: } x = -10, 10$$

Z.O.D.

$$(5x^2+500)^2 = 0$$
$$5x^2 + 500 = 0$$
$$5x^2 = -500$$
$$x^2 = -100$$
$$x = \text{DNE}$$

George should sell cookies for 10 days.

How many friends did he gain per dollar?

$$f(10) = \frac{2(10)}{5(10)^2 + 500}$$
$$= \frac{20}{5(100) + 500}$$
$$= \frac{20}{500 + 500}$$
$$f(10) = \frac{20}{1000}$$

$$f(10) = \frac{1}{5}\ f/\$$$

George gained $\frac{1}{5}$ of a friend per dollar.

Sentence answer:

To max friends per \$, George should sell cookies for 10 days and this will give him 1 friend for every \$5 he spends.