Advanced Techniques
6.2 – More Maximizing Applications

Ex A: Maximizing Tax Revenue

State Government Revenue

$t = tax\ rate$

$S(t) = total\ sales\ in\ country$

$Government\ revenue = S(t) \cdot t$

A nerd predict that the relationship between the tax rate *t* on an item and the total sales *S* of that item (in millions of dollars) is $s(t) = 9 - 20\sqrt{t}$ for $(0 \le t \le 0.20)$. Find the tax rate that maximizes revenue of the state government.

$GR(t) = S(t) \cdot t$
$= (9 - 20\sqrt{t})\,t$
$GR(t) = 9t - 20t^{3/2}$

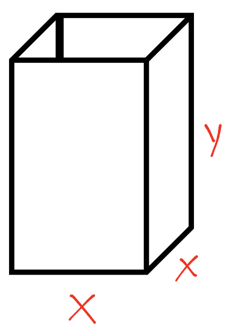
$GR'(t) = 9 - 30t^{1/2}$
$0 = 9 - 30\sqrt{t}$
$30\sqrt{t} = 9$
$\sqrt{t} = \frac{9}{30}$
$t = \left(\frac{9}{30}\right)^2$
$t = .09 = 9\%$

$GR(.09) = 9(.09) - 20(\sqrt{.09})^3$
$= .81 - 20(.3)^3$
$= .81 - 20(.027)$
$= .81 - .54$
$= \$.27$ million
$GR(.09) = \$270{,}000$

$GR''(t) = -15t^{-\frac{1}{2}}$
$GR''(.09) =$ neg, CCD, MAX

Ex B: Minimizing Package Materials

A moving company wishes to design an open-top box with a square base whose volume is exactly 32 cubic feet. Find the dimensions of the box requiring the least amount of materials.



$x =$ length of square base
$h =$ height
$V =$ volume $= 32\ ft^3$
$m =$ material $= SA =$ Surface Area

$SA = A_{bottom} + A_{left} + A_{Right} + A_{Front} + A_{Back}$
$= x^2 + xh + xh + xh + xh$
$SA = x^2 + 4xh$
$SA = x^2 + 4x\left(\frac{32}{x^2}\right)$
$SA = x^2 + \frac{128}{x}$
$M(x) = x^2 + 128x^{-1}$

$V = A_{base} \cdot h$
$32 = x^2 h$
$\frac{32}{x^2} = h$

$M'(x) = 2x - 128x^{-2}$
$0 = 2x - \frac{128}{x^2}$
$0 = 2x^3 - 128$
$128 = 2x^3$
$64 = x^3$
$4 = x$

$\frac{32}{(4)^2} = h$
$\frac{32}{16} = h$
$2 = h$

$M''(x) = x + 256x^{-3}$
$M''(4) =$ positive, CCU, MIN

$SA\underset{(h,x)}{(2,4)} = (4)^2 + 4(4)(2)$
$= 16 + 32$
$= 48\ ft^2$

To min the surface area to $48\ ft^2$, the square base should have dimensions of 4 ft and height of 2 feet.

Pro Tips

#1) Assign and define variables to each dimension of the box.

#2) Make equations for volume and surface area. Then use both of those equations to make a surface area function, *SA*.

#3) To minimize surface area, set $SA' = 0$ and solve. Make sure it is a min by using the second derivative test.

#4) Use *x* to answer the question asked.

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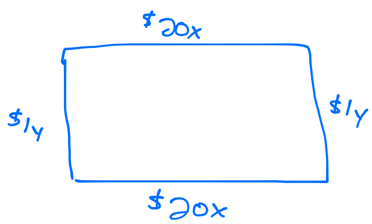
Ex C: Minimizing Cost

George wants to build a fence around his pet, Imagidragon. Imagidragon demands an area of 1000 square feet to frolic in the wind. The front and back of the dragon's play area will be a brick fence, priced at \$20 per linear foot. The other two sides will be fenced with a cotton balls costing \$1 per linear foot. Find the dimensions that will minimize the cost and find the minimum cost.



x = length of front & back

y = length of other two sides

$C_{(x,y)}$ = Cost function

A = Area

$$C_{(x,y)} = \$20x + \$20x + \$y + \$y$$

$$C_{(x,y)} = 40x + 2y$$

$$C(x) = 40x + 2\left(\frac{1000}{x}\right)$$

$$C(x) = 40x + 2000x^{-1}$$

$$A = (\text{length})(\text{width})$$

$$1000 = xy$$

$$\frac{1000}{x} = y$$

$$C'(x) = 40 - 2000x^{-2}$$

$$0 = 40 - \frac{2000}{x^2}$$

$$0 = 40x^2 - 2000$$

$$2000 = 40x^2$$

$$50 = x^2$$

$$\pm 7.07 \approx x$$

$$\frac{1000}{7.07} \approx y$$

$$141.44 \approx y$$

$$C''(x) = 4000x^{-3}$$

$$C''(-7.07) = \text{neg, CCD, MAX}$$

$$C''(7.07) = \text{pos, CCU, MIN}$$

$$C_{(7.07,\,141.44)} = 40(7.07) + 2(141.44)$$

$$\approx 282.8 + 282.88$$

$$C_{(7.07,\,141.44)} \approx \$565.68$$

To min the cost of the fence to \$565.68, George would make the front and back about 7.07 feet long and the other two sides about 141.44 ft long.

Pro Tips

#1) Assign and define variables to each dimension of the box.

#2) Make equations for area and cost. Then use both of those equations to make a cost function in one variable.

#3) To minimize cost, set $C(x)' = 0$ and solve. Make sure it is a min by using the second derivative test.

#4) Use x to answer the question asked.