Advanced Techniques
6.3A – Explicit vs Implicit Differentiation

For each equation, use implicit differentiation to find dy/dx.

#1) $y^3 - x^2 = 8$

$\frac{d}{dx}(y^3) - \frac{d}{dx}(x^2) = \frac{d}{dx}(8)$

$3y^2\frac{dy}{dx} - 2x = 0$

$3y^2\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{3y^2}$

#2) $x^3 = y^3 - 7$

$\frac{d}{dx}(x^3) = \frac{d}{dx}(y^3) - \frac{d}{dx}(7)$

$3x^2 = 3y^2\frac{dy}{dx} - 0$

$3x^2 = 3y^2\frac{dy}{dx}$

$\frac{3x^2}{3y^2} = \frac{dy}{dx}$

$\frac{x^2}{y^2} = \frac{dy}{dx}$

#3) $y^5 - x^4 = 5x$

$\frac{d}{dx}(y^5) - \frac{d}{dx}(x^4) = \frac{d}{dx}(5x)$

$5y^4\frac{dy}{dx} - 4x^3 = 5$

$5y^4\frac{dy}{dx} = 4x^3 + 5$

$\frac{dy}{dx} = \frac{4x^3+5}{5y^4}$

#4) $(x+2)^3 + (y+2)^3 = 21$

$\frac{d}{dx}(x+2)^3 + \frac{d}{dx}(y+2)^3 = \frac{d}{dx}(21)$

$3(x+2)^2\cdot\frac{d}{dx}(x+2) + 3(y+2)^2\frac{d}{dx}(y+2) = 0$

$3(x+2)^2(1) + 3(y+2)^2 1\cdot\frac{dy}{dx} = 0$

$3(y+2)^2\frac{dy}{dx} = -3(x+2)^2$

$\frac{dy}{dx} = \frac{-3(x+2)^2}{3(y+2)^2}$

$\frac{dy}{dx} = \frac{-(x+2)^2}{(y+2)^2}$

#5) $x^2y = 15$

$x^2 = 15y^{-1}$

$\frac{d}{dx}(x^2) = \frac{d}{dx}(15y^{-1})$

$2x = -15y^{-2}\frac{dy}{dx}$

$2x = \frac{-15}{y^2}\frac{dy}{dx}$

$\frac{2xy^2}{-15} = \frac{dy}{dx}$

#6) $xy - x = 9$

$x(y-1) = 9$

$y - 1 = 9x^{-1}$

$\frac{d}{dx}(y) - \frac{d}{dx}(1) = \frac{d}{dx}(9x^{-1})$

$1\cdot\frac{dy}{dx} - 0 = -9x^{-2}$

$\frac{dy}{dx} = \frac{-9}{x^2}$

#7) $x(y-1)^2 = 36$

$(y-1)^2 = 36x^{-1}$

$\frac{d}{dx}(y-1)^2 = \frac{d}{dx}(36x^{-1})$

$2(y-1)^1 \frac{d}{dx}(y-1) = -36x^{-2}$

$2(y-1)(1)\frac{dy}{dx} = \frac{-36}{x^2}$

$\frac{dy}{dx} = \frac{-36}{x^2(y-1)^2}$

$\frac{dy}{dx} = \frac{-18}{x^2(y-1)}$

#8) $y^3 - y^2 + y - 1 = x$

$\frac{d}{dx}(y^3) - \frac{d}{dx}(y^2) + \frac{d}{dx}(y) - \frac{d}{dx}(1) = \frac{d}{dx}(x)$

$3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} + 1 \cdot \frac{dy}{dx} - 0 = 1$

$\frac{dy}{dx}(3y^2 - 2y + 1) = 1$

$\frac{dy}{dx} = \frac{1}{3y^2 - 2y + 1}$

#9) $\frac{1}{x} + \frac{1}{y} = 22$

$\frac{d}{dx}(x^{-1}) + \frac{d}{dx}(y^{-1}) = \frac{d}{dx}(22)$

$-1x^{-2} - 1y^{-2}\frac{dy}{dx} = 0$

$\frac{-1}{x^2} - \frac{1}{y^2}\frac{dy}{dx} = 0$

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{1}{x^2}$

$\frac{dy}{dx} = \frac{-y^2}{x^2}$

#10) $x^3 = (y-2)^2 + 9$

$\frac{d}{dx}(x^3) = \frac{d}{dx}(y-2)^2 + \frac{d}{dx}(9)$

$3x^2 = 2(y-2)^1 \cdot \frac{d}{dx}(y-2) + 0$

$3x^2 = 2(y-2)\,1 \cdot \frac{dy}{dx}$

$\frac{3x^2}{2(y-2)} = \frac{dy}{dx}$

For each equation, find $\frac{dy}{dx}$ evaluated at the given value.

#11) $y^2 - x^3 = 1$ at (2, 3)

$\frac{d}{dx}(y^2) - \frac{d}{dx}(x^3) = \frac{d}{dx}(1)$

$2y\frac{dy}{dx} - 3x^2 = 0$

$2y\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{3(2)^2}{2(3)}$

$= \frac{3(4)}{6}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = 2$

#12) $y^2 = 6x^2 - 25$ at (1, 1)

$\frac{d}{dx}(y^2) = \frac{d}{dx}(6x^2) - \frac{d}{dx}(25)$

$2y\frac{dy}{dx} = 12x - 0$

$\frac{dy}{dx} = \frac{6x}{y}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{6(1)}{(1)}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = 6$

#13) $x^2y + xy^2 = 0$ at x = -2 and y = -1

$\frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) = \frac{d}{dx}(0)$

$\frac{d}{dx}(x^2)\cdot y + x^2\cdot\frac{d}{dx}(y) + \frac{d}{dx}(x)\cdot y^2 + x\cdot\frac{d}{dx}(y^2) = 0$

$2xy + x^2(1)\frac{dy}{dx} + (1)y^2 + x\cdot 2y\frac{dy}{dx} = 0$

$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$

$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$

$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$

$\left.\frac{dy}{dx}\right|_{(-2,-1)} = \frac{-2(-2)(-1) - (-1)^2}{(-2)^2 + 2(-2)(-1)}$

$= \frac{-4-1}{4+4}$

$\left.\frac{dy}{dx}\right|_{(-2,-1)} = -\frac{5}{8}$

#14) $x^2 + y^2 = xy + 6$ at (2, 3)

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(xy) + \frac{d}{dx}(6)$

$2x + 2y\frac{dy}{dx} = \frac{d}{dx}(x)\cdot y + x\cdot\frac{d}{dx}(y) + 0$

$2x + 2y\frac{dy}{dx} = y + x\frac{dy}{dx}$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y - 2x$

$\frac{dy}{dx}(2y - x) = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{(3)-2(2)}{2(3)-(2)}$

$= \frac{3-4}{6-2}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = -\frac{1}{4}$

For the demand equation, use implicit differentiation to find $\frac{dp}{dx}$

#15) $p^2 + p + 2x = 100$

$\frac{d}{dx}(p^2) + \frac{d}{dx}(p) + \frac{d}{dx}(2x) = \frac{d}{dx}(100)$

$2p\frac{dp}{dx} + 1\cdot\frac{dp}{dx} + 2 = 0$

$2p\frac{dp}{dx} + 1\frac{dp}{dx} = -2$

$\frac{dp}{dx}(2p+1) = -2$

$\frac{dp}{dx} = \frac{-2}{2p+1}$

#16) $12p^2 + 4p + 1 = x$

$\frac{d}{dx}(12p^2) + \frac{d}{dx}(4p) + \frac{d}{dx}(1) = \frac{d}{dx}(x)$

$24p\frac{dp}{dx} + 4\frac{dp}{dx} + 0 = 1$

$\frac{dp}{dx}(24p+4) = 1$

$\frac{dp}{dx} = \frac{1}{24p+4}$

#17) $xp^3 = 36$

$p^3 = 36x^{-1}$

$\frac{d}{dx}p^3 = \frac{d}{dx}36x^{-1}$

$3p^2\frac{dp}{dx} = -36x^{-2}$

$3p^2\frac{dp}{dx} = \frac{-36}{x^2}$

$\frac{dp}{dx} = \frac{-36}{3x^2p^2}$

$\frac{dp}{dx} = \frac{-12}{x^2p^2}$

#18) $(p + 5)(x + 2) = 120$

$\frac{d}{dx}\left[(p+5)(x+2)\right] = \frac{d}{dx}(120)$

$\frac{dp}{dx}(p+5)\cdot(x+2) + (p+5)\frac{d}{dx}(x+2) = 0$

$1\cdot\frac{dp}{dx}(x+2) + (p+5)(1) = 0$

$\frac{dp}{dx}(x+2) = -p-5$

$\frac{dp}{dx} = \frac{-p-5}{x+2}$