

Advanced Techniques

6.4A – Related Rates

Hint: the volume of a sphere exists ☺

Chocolate

#1) George left his large ball of chocolate in the sun so that its radius is decreasing at the rate of 2 inches per minute. How fast is the volume decreasing at the moment when the radius is 3 inches?

Sentence Answer:

Gallstones

#2) A gallstone is forming in George's gallbladder so that its radius is growing at the rate of 1 centimeter per year. How fast is its volume growing at the moment when the radius is 2 centimeters?

Sentence Answer:

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Gumball

#3) George is entering a chewed gumball contest. The radius of his spherical gumball is growing by $\frac{1}{2}$ centimeter per week. Find how rapidly the volume is increasing at the moment when the radius is 4 centimeters.

Sentence Answer:

Pig Feet

#4) George's profit from selling x boxes pig feet is $P = 1000x - \frac{1}{2}x^2$ dollars. If sales are growing at the rate of 20 per day, find how rapidly profit is growing (in dollars per day) when 600 boxes have been sold.

Sentence Answer:

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Tons of Feathers

#5) George's revenue from selling x tons of feathers is given as $R = 1000x - x^2$ dollars. If sales are increasing at the rate of 80 per day, find how rapidly revenue is growing (in dollars per day) when 400 tons have been sold.

Sentence Answer:

Accidents

#6) The number of traffic accidents George's stench causes per year in a population p is predicted to be $T = 0.002p^{3/2}$. If the population is growing by 500 people per year, find the rate at which traffic accidents will be rising when the population is 40,000.

Sentence Answer:

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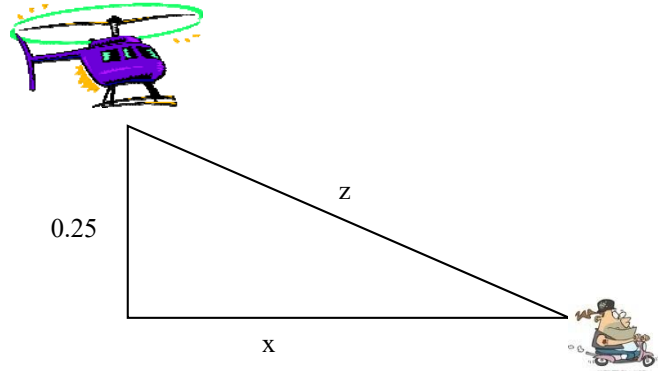
Carnival Slaying

#7) George is a carny and witnesses many types of crimes. The number of slayings at George's carnival of population p is expected to be $W = 0.003p^{4/3}$. If the population is growing by 1000 people per year, find the rate at which the number of carnival slayings will be increasing when the population is 1,000,000.

Sentence Answer:

Speeding

#8) A traffic patrol helicopter is stationary a quarter of a mile directly above a highway, as shown in the diagram below. Its radar detects George's moped whose line-of-sight distance from the helicopter is half a mile and is increasing at the rate of 57 mph. Is the moped exceeding the highway's speed limit of 60 mph?



Sentence Answer:

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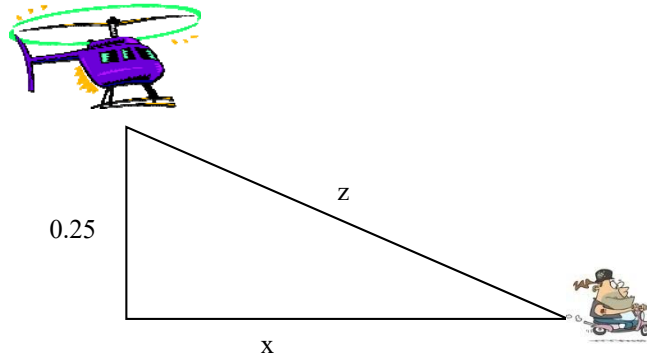
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- #1) At the moment the radius is 3 inches, the volume is decreasing by $72\pi \approx 226 \text{ in}^3$ in per hour.
- #2) When the radius is 2 mm, the volume is growing at $16\pi \approx 50.27 \text{ mm}^3$ per minute.
- #3) When the radius is 4 cm, the volume of the tumor is growing at $32\pi \approx 101 \text{ cm}^3$ per week.
- #4) When 600 units have been sold, the profit is growing by \$8000 per day.
- #5) When 400 units have been sold, the revenue is growing by \$16,000 per day.
- #6) When the population is 40,000 people, traffic accidents will be rising by 300 accidents per year.
- #7) When the population is 1,000,000 people, the number of carnival slayings is increasing by 400 cases per year.
- #8) The car is traveling at 65.8 mph, so yes the car is speeding.

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#8)



Facts:

z = line of sight distance in miles

x = horizontal distance in miles

$$\frac{dz}{dt} = 57 \text{ miles/hour}$$

Find $\frac{dx}{dt}$ evaluated at $z = .5$ miles

From Pythagoras

$$\begin{aligned} x^2 + 0.25^2 &= z^2 \\ x^2 + 0.25^2 &= 0.5^2 \\ x^2 + 0.0625 &= 0.25 \\ x^2 &= .1875 \\ x &\approx 0.433 \end{aligned}$$

From Pythagoras

$$x^2 + 0.25^2 = z^2$$

$$\frac{d}{dt} x^2 + \frac{d}{dt} 0.25^2 = \frac{d}{dt} z^2$$

Found $\frac{d}{dt}$

$$2x \frac{dz}{dt} + 0 = 2z \frac{dz}{dt}$$

Simplified

$$2x \frac{dz}{dt} = 2z \frac{dz}{dt}$$

Simplified

$$\frac{dz}{dt} = \frac{2z}{2x} \frac{dz}{dt}$$

Solving for $\frac{dz}{dt}$, I divided by $2x$

$$= \frac{z}{x} \frac{dz}{dt}$$

The 2's canceled, because $2 / 2 = 1$

$$\frac{dz}{dt} \text{ evaluated } \approx \frac{0.5}{.433} 57$$

Substituted $\frac{dz}{dt} = 57, z = .5, x \approx 0.433$

$$\approx \frac{0.5(57)}{.433}$$

Simplified

$$\approx 65.8 \text{ miles per hour}$$

Simplified

Yes, the moped is exceeding the highway's speed of 60 mph. George is traveling at a rate of approximately 65.8 mph.