Advanced Techniques
6.4 – Related Rates

Sometimes *both* variables in an equation will be functions of a *third* variable, usually t for time. For example, for a seasonal product such as fur coats, the price p and weekly sales x will be related by a demand equation, and both price p and quantity x will depend on the time of year.

Differentiating both sides of the demand equation with respect to time t will give an equation relating the derivatives $\frac{dp}{dt}$ and $\frac{dx}{dt}$

Such "related rates" equations show how fast one quantity is changing relative to another.

Ex A: Finding Related Rates

#1) A pebble thrown into a pond causes circular ripples to radiate outward. If the radius of the outer ripple is growing by 2 feet per second, how fast is the area of the circle growing at the moment when the radius is 10 feet?

Since the radius is increasing by 2 feet per second, $\frac{dr}{dt} =$

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#2) A boat yard's total profit from selling x outboard motors is $P(x) = -x^2 + 1000x - 2000$. If the outboards are selling at the rate of 20 per week, how fast is the profit changing when 400 motors have been sold?

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#3) A study of urban pollution predicts that sulfur oxide emissions in a city will be $S(x) = 0.1x^2 + 20x + 2$ tons, where x is the population (in thousands). The population of the city t years from now is expected to be $x = 20\sqrt{t} + 800$ thousand people. Find how rapidly sulfur oxide pollution will be increasing 4 years from now.

Pro Tips