# Basic Integration

## 7.1A – Antiderivatives & Indefinite Integrals

A: Find each indefinite integral.

#1) $\int x^5 dx = \frac{1}{6}x^6 + C$

#2) $\int x^6\, dx = \frac{1}{7}x^7 + C$

#3) $\int t^{1/2}\, dt = \frac{2}{3}t^{3/2} + C$

#4) $\int t^{5/4}\, dt = \frac{4}{9}t^{9/4} + C$

#5) $\int \sqrt{u^7}\, du = \int u^{7/2} du$

$= \frac{2}{9}u^{9/2} + C$

$= \frac{2}{9}\sqrt{u^9} + C$

#6) $\int \sqrt{u^3}\, du = \int u^{3/2} du$

$= \frac{2}{5}u^{5/2} + C$

#7) $\int \frac{dw}{\sqrt{w}} = \int w^{-1/2} dw$

$= 2w^{1/2} + C$

#8) $\int \frac{dw}{w^2} = \int w^{-2} dw$

$= -1w^{-1} + C$

$= \frac{-1}{w} + C$

#9) $\int (t^5 - t^3) dt = \frac{1}{6}t^6 - \frac{1}{4}t^4 + C$

#10) $\int (t^2 + t) dt = \frac{1}{3}t^3 + \frac{1}{2}t^2 + C$

#11) $\int(x^{1/2} - x^{11})dx$

$= \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{12}x^{12} + C$

#12) $\int(x^{7/2} + x^5)\,dx$

$= \frac{2}{9}x^{\frac{9}{2}} + \frac{1}{6}x^6 + C$

#13) $\int(w^9 + w^{-5} + w)dw$

$= \frac{1}{10}w^{10} - \frac{1}{4}w^{-4} + \frac{1}{2}w^2 + C$

#14) $\int(w^{-2/3} - w^{-6} + w^{-2})\,dw$

$= 3w^{\frac{1}{3}} + \frac{1}{5}w^{-5} - w^{-1} + C$

#15) $\int 8x^7\,dx$

$= \frac{1}{8}x^8 + C$

#16) $\int 32x^{31}\,dx$

$= x^{32} + C$

#17) $\int 2u^5\,du$

$= \frac{1}{3}u^6 + C$

#18) $\int \frac{1}{3}u^8 du$

$= \frac{1}{27}u^9 + C$

#19) $\int 11\,dz$

$= 11z + C$

#20) $\int -23\,dz$

$= -23z + C$

#21) $\int 103\, dv$

$= 103v + C$

#22) $\int dv$

$= v + C$

#23) $\int (8x + 11)\, dx$

$= 4x^2 + 11x + C$

#24) $\int (5x^4 + 2x)\, dx$

$= x^5 + x^2 + C$

#25) $\int \left(8\sqrt{t^5} - \frac{1}{t^{1/3}}\right) dt$

$= \int \left(8t^{5/2} - t^{-1/3}\right) dt$

$= \frac{16}{7} t^{7/2} - \frac{3}{2} t^{2/3} + C$

#26) $\int \left(15t^{2/3} - \frac{15}{t^{2/3}}\right) dt$

$= \int \left(15t^{2/3} - 15t^{-2/3}\right) dt$

$= 15\left(\frac{3}{5}\right)t^{5/3} - 15(3)t^{1/3} + C$

$= 9t^{5/3} - 45t^{1/3} + C$

#27) $\int \left(14\sqrt{w^5} - \frac{12}{w^{1/5}}\right) dw$

$= \int \left(14w^{1/5} - 12w^{-1/5}\right) dw$

$= 14\left(\frac{5}{6}\right)w^{6/5} - 12\left(\frac{5}{4}\right)w^{4/5} + C$

$= \frac{35}{3}\sqrt[5]{w^6} - 15\sqrt[5]{w^4} + C$

#28) $\int \left(20w^{2/3} + \frac{1}{w^{2/3}}\right) dw$

$= \int \left(20w^{2/3} + w^{-2/3}\right) dw$

$= 20\left(\frac{3}{5}\right)w^{5/3} - 3w^{1/3} + C$

$= 12w^{5/3} - 3w^{1/3} + C$

#29) $\int (x - 1)^2 dx$

$= \int (x^2 - 2x + 1)\, dx$

$= \frac{1}{3}x^3 - x^2 + x + C$

#30) $\int (x - 1)(x + 1) dx$

$= \int (x^2 - 1)\, dx$

$= \frac{1}{3}x^3 - x + C$

#31) $\int t^5(t+7)\,dt$

$= \int (t^6 + 7t^5)\,dt$

$= \frac{1}{7}t^7 + \frac{7}{6}t^6 + C$

#32) $\int \sqrt{t}(2t^2 + 8t^6)\,dt$

$= \int t^{\frac{1}{2}}(2t^2 + 8t^6)\,dt$

$= \int \left(2t^{\frac{5}{2}} + 8t^{\frac{13}{2}}\right)dt$

$= 2\left(\frac{2}{7}\right)t^{\frac{7}{2}} + 8\left(\frac{2}{15}\right)t^{15/2} + C$

$= \frac{4}{7}\sqrt{t^7} + \frac{16}{15}\sqrt{t^{15}} + C$

#33) $\int (z^2 - 9)(z^2 + 9)\,dz$

$= \int (z^4 - 81)\,dz$

$= \frac{1}{5}z^5 - 81x + C$

#34) $\int \frac{9z^9 - 9z^8 + z^7}{z^7}\,dz$

$= \int \frac{z^7(9z^2 - 9z + 1)}{z^7}\,dz$

$= \int (9z^2 - z + 1)\,dz$

$= 3z^3 - \frac{1}{2}z^2 + z + C$

#35) $\int \frac{r^2 - 100}{r - 10}\,dr$

$= \int \frac{(r-10)(r+10)}{r-10}\,dr$

$= \int (r+10)\,dr$

$= \frac{1}{2}r^2 + 10r + C$

#36) $\int \frac{r^2 + 12r + 36}{r + 6}\,dr$

$= \int \frac{(r+6)^2}{r+6}\,dr$

$= \int (r+6)\,dr$

$= \frac{1}{2}r^2 + 6r + C$

#37) $\int \frac{w+5}{w^3 + 5w^2}\,dw$

$= \int \frac{w+5}{w^2(w+5)}\,dw$

$= \int \frac{1}{w^2}\,dw$

$= \int w^{-2}\,dw$

$= -1w^{-1} + C$

$= \frac{-1}{w} + C$

**Wedgies R Us**

#38) After figuratively and literally burning Castle Greyskull to the ground, George gets another grand idea: selling wedgies. He forms a new company, Wedgies R Us, and inexplicitly knows the marginal cost function is $MC(x) = \frac{200}{\sqrt{x}}$ where $x$ is the number of wedgies performed, and fixed costs are \$1. Find the cost function.

| $C(x) = \text{cost}$ | $x = \text{wegies}$ | $(0, \$1)$ |
|---|---|---|

$MC(x) = 200x^{-\frac{1}{2}}$

$C(x) = \int MC(x)\,dx$

$= \int 200x^{-\frac{1}{2}}\,dx$

$= 400x^{\frac{1}{2}} + C$

$C(x) = 400\sqrt{x} + C$

@(0,1)

$1 = 400\sqrt{0} + C$

$1 = C$

$C(x) = 400\sqrt{x} + 1$

**Wheel Chairs**

#39) After several disastrous demonstrations of his deluxe line of wedgies, George decides to sell celebrity endorsed wheel chairs. With the aid of Christopher Reeves, he forms *If You Cant Fly, You Might As Well Roll, Inc*. He somehow decides his marginal profit function is $MP(x) = 0.03\sqrt{x}$ where $x$ is the number of wheel chairs sold. The company is operating at a loss of \$5000 when it has sold no no wheel chairs. Find the profit function.

| $P(x) = \text{Profit}$ | $x = \text{wheel chairs}$ | $(0, -5000)$ |
|---|---|---|

$MP(x) = 0.03x^{\frac{1}{2}}$

$P(x) = \int MP(x)\,dx$

$= \int 0.03x^{\frac{1}{2}}\,dx$

$= 0.03\left(\frac{2}{3}\right)x^{\frac{3}{2}} + C$

$P(x) = 0.02\sqrt{x^3} + C$

@(0,-5000)

$-5000 = 0.02\sqrt{(0)^3} + C$

$-5000 = C$

$P(x) = 0.02\sqrt{x^3} - 5000$

**Free Falling**

#40) Being a constant confuser of reality and fantasy, George thinks Christopher Reeves is actually Superman and is pretending to be paralyzed just to give Lex Luther a false sense of security. While filming their first commercial, George and Christopher are on the Skydeck of the Sears Tower when George shoves Christopher Reeves off the building, hoping to see him fly in person. While falling, Christopher's velocity is $v(t) = 32t$ feet per second traveling down after *t* seconds.

a. Find a formula for distance traveled after *t* seconds.
b. Find the distance that Christopher falls in the first 5 seconds.
c. How long would it take for Reeves to hit the ground? (Hint: I won't give you one.)

**Fortunate Charms**

#41) Wanting to see an epic duel, George tricked Gene Hackman (who George thought was really Lex Luthor) into hanging out at the bottom of Sears Tower. Right before pushing Christopher Reeves off the Skydeck, George text Gene, "What's *up*?" Upon reading the text, Gene looked up seeing a plunging Christopher Reeves. Acting on instinct, Gene Hackman began making a pillow of marshmallows out of the industrial sized vat of Lucky Charms he had just purchased. He made a pillow of charms at a rate of $C(t) = 6t^{-1/2}$ charms per second, where *t* is time in seconds.

a. Find a function *TC(t)* for the total charms counted in *t* seconds.
b. Find the number of Lucky Charms laid in 9.2 seconds.

**Squirrels Vs Bears**

#42) While waiting for Superm…, I mean, Christopher Reeves to fly, George got bored and began to play with his toy squirrels and bears. (Read: carcasses) The Bears decided to exterminate the Squirrels via pollutants. Before the Bears' evil plan to poison the Squirrels, Squirrel Village contained 20 ounces of pollution. Evil Bear Corp's chemical plant is adding pollution to Squirrel Village at a rate of $r(t) = 40t^{1/3}$ ounces per week, were $t$ is the number of weeks the Evil Bear Corp has been trying to smoke out the Squirrels.

a. Find the function, *P(t)* for the total amount of pollution in the first $t$ weeks.
b. Find how much pollution will enter Squirrel Village the first 8 weeks.
c. If 2500 ounces of pollution will destroy all the squirrels, will the village survive 27 weeks?

Answers

#1) $\frac{1}{6}x^6 + C$
#2) $\frac{1}{7}x^7 + C$
#3) $\frac{2}{3}t^{3/2} + C$
#4) $\frac{4}{9}t^{9/4} + C$
#5) $\frac{2}{9}u^{9/2} + C$
#6) $\frac{2}{5}u^{5/2} + C$
#7) $2w^{1/2} + C$

#8) $\frac{-1}{w} + C$
#9) $\frac{1}{6}t^6 - \frac{1}{4}t^4 + C$
#10) $\frac{1}{3}t^3 + \frac{1}{2}t^2 + C$
#11) $\frac{2}{3}x^{3/2} - \frac{1}{12}x^{12} + C$
#12) $\frac{2}{9}x^{9/2} + \frac{1}{6}x^6 + C$
#13) $\frac{1}{10}w^{10} - \frac{1}{4}w^{-4} + \frac{1}{2}w^2 + C$
#14) $3w^{1/3} + \frac{1}{5}w^{-5} - w^{-1} + C$

#15) $x^8 + C$
#16) $x^{32} + C$
#17) $\frac{1}{3}u^6 + C$
#18) $\frac{1}{27}u^9 + C$
#19) $11z + C$
#20) $-23z + C$
#21) $103v + C$

#22) $v + C$
#23) $4x^2 + 11x + C$
#24) $x^5 + x^2 + C$
#25) $\frac{16}{7}t^{7/2} - \frac{3}{2}t^{2/3} + C$
#26) $9t^{5/3} - 45t^{1/3} + C$
#27) $4w^{7/2} - 15w^{4/5} + C$

#28) $12w^{5/3} + 3w^{1/3} + C$
#29) $\frac{1}{3}x^3 - x^2 + x + C$
#30) $\frac{1}{3}x^3 - x + C$
#31) $\frac{1}{7}t^7 + \frac{7}{6}t^6 + C$
#32) $\frac{4}{7}t^{7/2} + \frac{16}{15}t^{15/2} + C$

#33) $\frac{1}{5}z^5 - 81z + C$
#34) $3z^3 - \frac{9}{2}z^2 + z + C$
#35) $\frac{1}{2}r^2 + 10r + C$
#36) $\frac{1}{2}r^2 + 6r + C$
#37) $\frac{-1}{w} + C$

#38) $C(x) = 400\sqrt{x} + 1$
#39) $P(x) = 0.02x^{\frac{3}{2}} - 5000$
#40a) $d(t) = 16t^2$
#40b) 400 feet
#40c) about 9.2 seconds
#41a) $TC(t) = 12\sqrt{t}$
#41b) about 36 charms
#42a) $P(t) = 30t^{\frac{4}{3}} + 20$
#42b) 500 ounces
#42c) Yes, but they'll have less than a week left