Basic Integration
7.1 – Antiderivatives and Indefinite Integrals

Introduction

Antidifferentiation and Integration are synonyms

Everything a derivative does, antidifferentiation does backwards.

Examples:

$v'(t) = a(t)$	$\int a(t)\,dt = v(t)$
$s'(t) = v(t)$	$\int v(t)\,dt = s(t)$
$\text{total}' = \text{rate}$	$\int \text{rate}\,dx = \text{total}$

Antiderivatives and Indefinite Integrals

If $f'(x) = 2x$ then what is $f(x)$?

$f(x) = x^2 + C$

$f(x) = x^n$	$f'(x) = x^n$
$f'(x) = n \cdot x^{n-1}$	$f(x) = \frac{1}{n+1} x^{n+1}$

Indefinite Integral

$$\int f(x)dx = g(x) + C$$

Integral Sign — Integrand — Arbitrary Constant

The integral of f(x) is g(x) + C if and only if $g'(x) = f(x)$ the derivative of g(x) is f(x)

Integrand: The function to be integrated.

dx: Reminds us that the variable of integration is x.

Constant, C: It is an arbitrary constant since it can take any value, positive, negative, or zero.

Power rule for Integration

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$$

Sum Rule for Integration

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

Constant Multiple Rule for Integration

$$\int k \cdot f(x)dx = k\int f(x)dx$$

Integral of a Constant

$$\int kdx = kx + C$$

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Ex. A: Finding an Indefinite Integral (Check your answer by differentiation.)

#1) $\int x^2 dx$

$= \frac{1}{3}x^3 + C$

#2) $\int t^{10} dt$

$= \frac{1}{11}t^{11} + C$

#3) $\int \sqrt{z} dz$

$= \int z^{\frac{1}{2}} dz$

$= \frac{2}{3}z^{3/2} + C$

$= \frac{2}{3}\sqrt{z^3} + C$

#4) $\int \frac{1}{u^2} du$

$= \int u^{-2} du$

$= -1u^{-1} + C$

$= \frac{-1}{u} + C$

#5) $\int \frac{dx}{x^4}$

$= \int \frac{1}{x^4} dx$

$= \int x^{-4} dx$

$= -\frac{1}{3}x^{-3} + C$

$= \frac{-1}{3x^3} + C$

#6) $\int 1 dt$

$= \int 1t^0 dt$

$= 1 \cdot t^1 + C$

$= t + C$

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#7) $\int z^2 dz$

$= \frac{1}{3}z^3 + C$

#8) $\int u^{-2} du$

$= -1u^{-1} + C$

$= -u^{-1} + C$

#9) $\int x^{-2/3} dx$

$= 3x^{1/3} + C$

#10) $\int x^{-1} dx$

$= \frac{1}{0}x^0$

$=$ undefined

$\int x^{-1} dx =$ undefined

#11) $\int (x^2 + x^5) dx$

$= \int x^2 dx + \int x^5 dx$

$= \frac{1}{3}x^3 + \frac{1}{6}x^6 + C$

#12) $\int 6x^2 dx$

$= 6 \int x^2 dx$

$= 6 \cdot \frac{1}{3}x^3 + C$

$= 2x^3 + C$

#13) $\int 7dx$

$= 7x + C$

#14) $\int (5x^2 - 2x^{-2} + 5)dx$

$= 5\int x^2 dx - 2\int x^{-2} dx + \int 5 dx$

$= 5 \cdot \frac{1}{3}x^3 - 2(-1)x^{-1} + 5x + C$

$= \frac{5}{3}x^3 + 2x^{-1} + 5x + C$

#15) $\int \left(\frac{3\sqrt{x}}{5} - \frac{5}{\sqrt{x}}\right) dx$

$= \frac{3}{5}\int x^{\frac{1}{2}} dx - 5\int x^{-\frac{1}{2}} dx$

$= \frac{3}{5}\left(\frac{2}{3}\right)x^{3/2} - 5(2)x^{\frac{1}{2}} + C$

$= \frac{2}{5}\sqrt{x^3} - 10\sqrt{x} + C$

#16) $\int \left(\sqrt[4]{w} - \frac{5}{w^4}\right) dw$

$= \int w^{\frac{1}{4}} dw - 5\int w^{-4} dw$

$= \frac{4}{5}w^{5/4} - 5\left(-\frac{1}{3}\right)w^{-3} + C$

$= \frac{4}{5}\sqrt[4]{w^5} + \frac{5}{3w^3} + C$

#17) $\int 10t^9 dt$

$= t^{10} + C$

#18) $\int 4x^3 dx$

$= x^4 + C$

#19) $\int 3u^2 du$

$= u^3 + C$

#20) $\int 8z^7 dz$

$= z^8 + C$

#21) $\int [x^2(x+5)^2] dx$

$= \int [x^2(x^2+10x+25)] dx$

$= \int [x^4+10x^3+25x^2] dx$

$= \frac{1}{5}x^5 + 10\left(\frac{1}{4}\right)x^4 + 25\left(\frac{1}{3}\right)x^3 + C$

$= \frac{1}{5}x^5 + \frac{5}{2}x^4 + \frac{25}{3}x^3 + C$

#22) $\int \frac{3x^2-x}{x} dx$

$= \int \frac{x(3x-1)}{x} dx$

$= \int (3x-1) dx$

$= 3\cdot\frac{1}{2}x^2 - x + C$

$= \frac{3}{2}x^2 - x + C$

Since differentiation turns a cost function into a marginal cost function, integration turns a marginal cost function back into a cost function. To evaluate the constant, however, we need the fixed costs.

Finding the cost function involves two steps:
#1: Integrate the marginal cost to find the cost function.
#2: Use the fixed cost to evaluate the arbitrary constant.

Dumpster Diving
#1) While dumpster diving George fantasizes about owing his very own He-Man collection. Closing his eyes, George imagines living in Castle Greyskull where he and He-Man would be the bestest of friends. Expanding the boundaries of their relationship, George and He-Man would team up to sell Power Swords. The marginal cost function for producing Power Swords is $MC(x) = 6\sqrt{x}$ and the fixed cost is \$1000. Find the cost function.

x = # of power Swords produced.	C(x) = total cost	(0, \$1000)

$$C(x) = \int MC(x)\,dx$$
$$= \int 6x^{\frac{1}{2}}\,dx$$
$$= 6\left(\tfrac{2}{3}\right)x^{\frac{3}{2}} + C$$
$$C(x) = 4\sqrt{x^3} + C$$

@ (0,1000)
$$1000 = 4\sqrt{(0)^3} + C$$
$$1000 = C$$

$$C(x) = 4\sqrt{x^3} + 1000$$

Castle Greyskull, LLC
#2) As is often the case, George's fantasy gets mixed with reality. George's delusions of grandeur lead him to form an LLC. While filling out his paperwork with the state of Ohio for Castle Greyskull, LLC, George claims the current net worth of Castle Greyskull, LLC is \$78 billion and will grow at the rate of $4.4t^{-1/3}$ billion dollars per year *t* years from now. Find a formula for the net worth of Castle Greyskull, LLC after t years. Then use your formula to find the net worth after 8 years.

t = years from now	NW = net worth (\$ billions) (0, \$78 billion)

$$NW = \int NW'\,dt$$
$$= \int 4.4t^{-\frac{1}{3}}\,dt$$
$$= \overset{2.2}{\cancel{4.4}}\left(\tfrac{3}{2}\right)t^{2/3} + C$$
$$NW = 6.6t^{2/3} + C$$

@ (0,78)
$$78 = 6.6(0)^{2/3} + C$$
$$78 = C$$

$$NW(t) = 6.6t^{2/3} + 78$$

$$NW(8) = 6.6\left(\sqrt[3]{(8)}\right)^2 + 78$$
$$= 6.6(2)^2 + 78$$
$$= 6.6(4) + 78$$
$$= 26.4 + 78$$
$$NW(8) = \$104.4 \text{ billion}$$