Basic Integration

7.2 – Integration Using Logarithmic & Exponential Functions

What does $\frac{d}{dx}\ln(x) = \frac{x'}{x} = \frac{1}{x}$

What does $\frac{d}{dx}\ln(-x) = \frac{(-x)'}{-x} = \frac{-1}{-x} = \frac{1}{x}$

The Integral $\int \frac{1}{x} dx$

$$\int \frac{1}{x} dx = ln|x| + C$$

The Integral $\int e^{ax} dx$

For any constant $a \neq 0$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

Ex A: Integrate

#1) $\int \frac{6}{x} dx = 6\int \frac{1}{x} dx$

$= 6 \ln|x| + C$

#2) $\int (x^{-1} - x^{-2} + x^2) dx = \int \left(\frac{1}{x} - x^{-2} + x^2\right) dx$

$= \ln|x| - (-1)x^{-1} + \frac{1}{3}x^3 + C$

$= \ln|x| + x^{-1} + \frac{1}{3}x^3 + C$

#3) $\int \frac{5}{2x} dx = \frac{5}{2} \ln|x| + C$

#4) $\int e^{3x} dx = \frac{1}{3} e^{3x} + C$

#5) $\int 10e^{-5x} dx = 10\left(-\frac{1}{5}\right)e^{-5x} + C$

$= -2e^{-5x} + C$

#6) $\int e^{x} dx = e^{x} + C$

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In applications, to evaluate the constant C :
1. Evaluate the integral at the sometimes hidden ordered pair
2. Solve for C.
3. Write the answer with C replaced by its correct value.

Carcassitis

#1) After playing with animal carcasses, George becomes ill with a contagious disease called carcassitis. While licking handrails and deli counters, his illness sweeps across Chicago and spreads at the rate of $12e^{0.2t}$ new cases per day, where t is the number of days since the epidemic began. The epidemic began with 1 case, obviously.

t = # of days since epidemic began

T = total cases

a. Find a formula for the total number of carcassitis' cases in the first t days of the epidemic.

(0,1)

$$T = \int 12e^{0.2t}\,dt$$
$$= 12(5)e^{0.2t} + C$$
$$T = 60e^{0.2t} + C$$
$$1 = 60e^{0.2(0)} + C$$
$$1 = 60 + C$$
$$-59 = C$$
$$T(t) = 60e^{0.2t} - 59$$

b. Use your formula to find the number of carcassitis cases during the first 30 days.

$$T(30) = 60e^{0.2(30)} - 59$$
$$= 60e^{6} - 59$$
$$T(30) \approx 24{,}147 \text{ cases}$$

There will be a total of 24,147 cases the first 30 days

Antidote

#2) Seeing an opportunity to make some quick cash, George develops an antidote *(read: sewer water infused with ice cream sprinkles)* to carcassitis. He estimates that during day t of carcassitis, the antidote will sell at a rate of approximately $25/t$ per day, where $t = 1$ corresponds to the beginning of the sale, at which time none have been sold. Find a formula for the total number of antidotes that will be sold up to day t. Will the George's inventory of 64 serums be sold by day $t = 12$?

t = day of sale

A = antidotes sold

(1,0)

$$A = \int \frac{25}{t}\,dt$$
$$A = 25\ln|t| + C$$
$$0 = 25\ln|1| + C$$
$$0 = 0 + C$$
$$0 = C$$
$$A(t) = 25\ln|t|$$

$$A(12) = 25\ln|12|$$
$$A(12) \approx 62 \text{ serums}$$

George would only sell 62 of his serums so he will have 2 left.

CRAP

#3) Trying to raise money to develop more antidote, George forms the Carcassitis Radiation Association Program where he is both the president and the only employee. He tells prospective investors that CRAP predicts the annual consumption of antidotes will be $0.23e^{0.01t}$ million metric tons per year, where t is the number of years since 2015. Find a formula for the total antidote consumption within t years of 2015 and estimate when the known world reserves of 7 million metric tons will be exhausted.

(0,0)

t = years after 2015

A = annual consumption of Crap (millions of metric tons)

$$A = \int 0.23e^{0.01t}\,dt$$
$$= 0.23(100)e^{0.01t} + C$$
$$A = 23e^{0.01t} + C$$
$$0 = 23e^{0.01(0)} + C$$
$$0 = 23e^{0} + C$$
$$0 = 23 + C$$
$$-23 = C$$
$$A = 23e^{0.01t} - 23$$

$$A = 23e^{0.01t} - 23$$
$$7 = 23e^{0.01t} - 23$$
$$30 = 23e^{0.01t}$$
$$\frac{30}{23} = e^{0.01t}$$
$$\ln\frac{30}{23} = \ln e^{0.01t}$$
$$\ln\frac{30}{23} = 0.01t$$
$$100\ln\frac{30}{23} = t$$
$$26.6 \approx t$$

The world reserve of CRAP will be exhausted during the year 2041