

Basic Integration

7.3 – Integration With Trigonometry

Anti-Derivatives of the Six Trigonometric Functions

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \csc(x) dx = \ln|\csc(x) - \cot(x)| + C$$

$$\int \tan(x) dx = \ln|\sec(x)| + C$$

$$\int \cot(x) dx = \ln|\sin(x)| + C$$

More Anti-Derivatives

$$\int \csc(x) \cot(x) dx = -\csc(x) + C$$

$$\int \sec(x) \tan(x) dx = \sec(x) + C$$

$$\int \sec^2(x) dx = \tan(x) + C$$

$$\int \csc^2(x) dx = -\cot(x) + C$$

Reciprocal Identities

$$\sin(x) = \frac{1}{\csc(x)}$$

$$\cos(x) = \frac{1}{\sec(x)}$$

$$\tan(x) = \frac{1}{\cot(x)}$$

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)}$$

Quotient Identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities:

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

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Ex A: Integrate each function

$$\begin{aligned}\#1) \quad & \int [2 \sin(x) + \cos(x)] dx \\ & = -2 \cos(x) + \sin(x) + C\end{aligned}$$

$$\begin{aligned}\#2) \quad & \int [\csc(x)(\cot(x) - \csc(x))] dx \\ & = \int [C \csc(x) \cot(x) - \csc^2(x)] dx \\ & = -C \csc(x) - (-\cot(x)) + C \\ & = -\csc(x) + \cot(x) + C\end{aligned}$$

$$\begin{aligned}\#3) \quad & \int \sqrt{\pi \cos^2(x)} dx \\ & = \int \sqrt{\pi} \cos(x) dx \\ & = \sqrt{\pi} \sin(x) + C\end{aligned}$$

$$\begin{aligned}\#4) \quad & \int \sqrt{1 - \cos^2(x)} dx \\ & = \int \sqrt{\sin^2(x)} dx \\ & = \int \sin(x) dx \\ & = -\cos(x) + C\end{aligned}$$

$$\begin{aligned}\#5) \quad & \int \cos(x) \sec(x) dx \\ & = \int 1 dx \\ & = x + C\end{aligned}$$

$$\begin{aligned}\#6) \quad & \int \sec^2(x) \cot^2(x) dx \\ & = \int \frac{1}{\cos^2(x)} \cdot \frac{\cos^2(x)}{\sin^2(x)} dx \\ & = \int \frac{1}{\sin^2(x)} dx \\ & = \int \csc^2(x) dx \\ & = -\cot(x) + C\end{aligned}$$