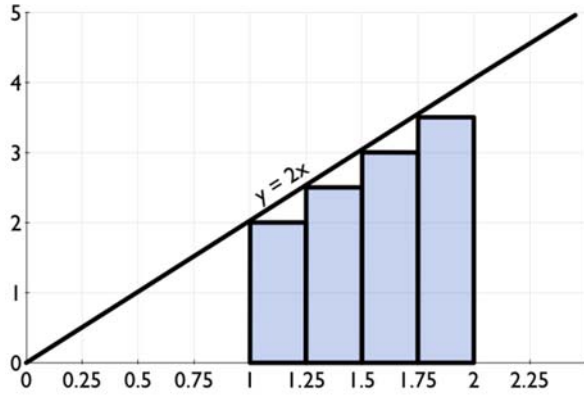


Basic Integration

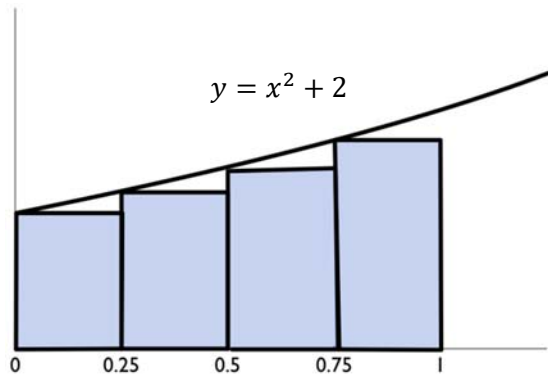
7.4A – Definite Integrals & Riemann Sum

A: Use the graphs to find each area

#1) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.



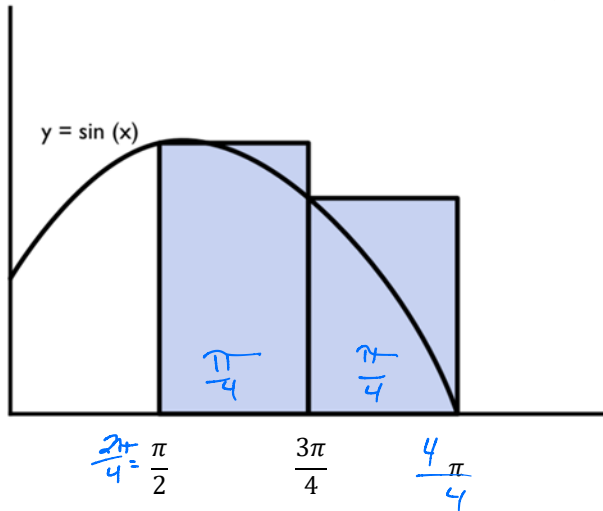
#2) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.



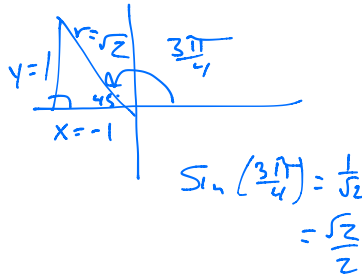
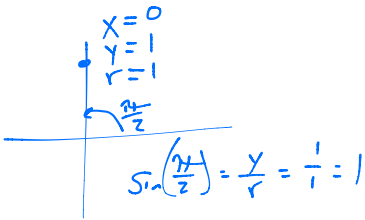
Basic Integration

7.4A – Definite Integrals & Riemann Sum

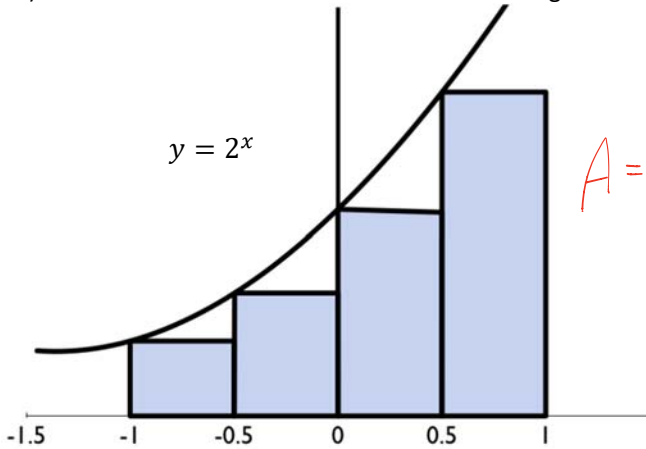
#3) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.



$$\begin{aligned} \Delta x &= \frac{\pi}{4} \\ A &= \int_{\frac{\pi}{2}}^{\pi} \sin(x) dx \approx f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x \\ &\approx f\left(\frac{\pi}{2}\right) \cdot \frac{\pi}{4} + f\left(\frac{3\pi}{4}\right) \cdot \frac{\pi}{4} \\ &= \sin\left(\frac{\pi}{2}\right) \cdot \frac{\pi}{4} + \sin\left(\frac{3\pi}{4}\right) \cdot \frac{\pi}{4} \\ &\approx 1 \cdot \frac{\pi}{4} + \frac{\sqrt{2}}{2} \cdot \frac{\pi}{4} \\ &\approx \frac{2\pi}{8} + \frac{\pi\sqrt{2}}{8} \\ &\approx \frac{2\pi + \pi\sqrt{2}}{8} \\ &\approx 1.34 \text{ un}^2 \end{aligned}$$



#4) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.

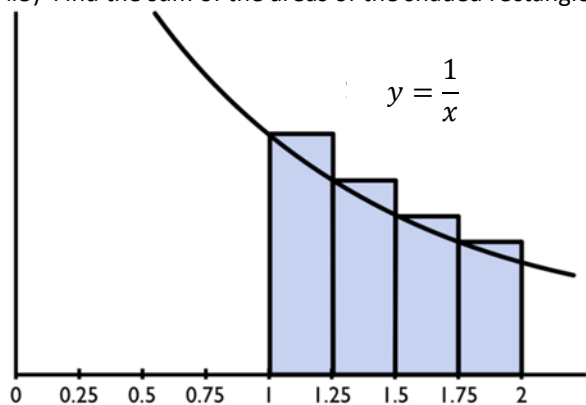


$$\begin{aligned} \Delta x &= 0.5 \\ A &= \int_{-1}^1 2^x dx \approx f(x_1) \cdot \Delta x + f(x_2) \cdot \Delta x + f(x_3) \cdot \Delta x + f(x_4) \cdot \Delta x \\ &\approx f(-1) \cdot (0.5) + f(-0.5) \cdot (0.5) + f(0) \cdot (0.5) + f(0.5) \cdot (0.5) \\ &\approx 2^{-1} \cdot \frac{1}{2} + 2^{-0.5} \cdot \frac{1}{2} + 2^0 \cdot \frac{1}{2} + 2^{0.5} \cdot \frac{1}{2} \\ &\approx \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + \sqrt{2} \cdot \frac{1}{2} \\ &\approx \frac{1 \cdot \sqrt{2}}{4\sqrt{2}} + \frac{1 \cdot 2}{2\sqrt{2} \cdot 2} + \frac{1 \cdot 2\sqrt{2}}{2\sqrt{2}} + \frac{\sqrt{2} \cdot 2\sqrt{2}}{2 \cdot \sqrt{2}} \\ &\approx \frac{\sqrt{2}}{4\sqrt{2}} + \frac{2}{4\sqrt{2}} + \frac{2\sqrt{2}}{4\sqrt{2}} + \frac{4}{4\sqrt{2}} \\ &\approx \frac{6 + 3\sqrt{2}}{4\sqrt{2}} \\ &\approx 1.81 \text{ un}^2 \end{aligned}$$

Basic Integration

7.4A – Definite Integrals & Riemann Sum

#5) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.



B: Approximate the area under the curve from a to b by calculating a Riemann sum with the given number of rectangles.

#6) $f(x) = 5x$, $a = 1$, $b = 2$, $n = 5$

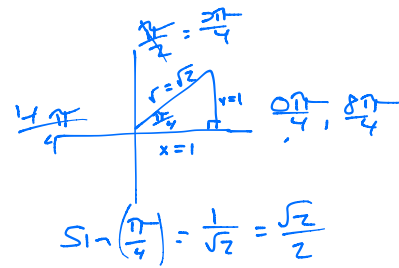
Basic Integration

7.4A – Definite Integrals & Riemann Sum

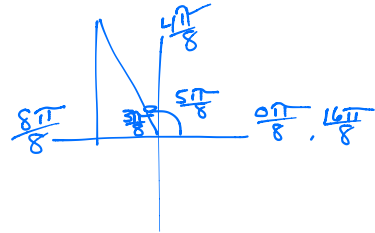
$$\frac{2\pi}{8} + \frac{3\pi}{8} = \frac{5\pi}{8}$$

#7) $f(x) = \sin(x)$, $a = \frac{\pi}{4}$, $b = \pi$, $n = 2$

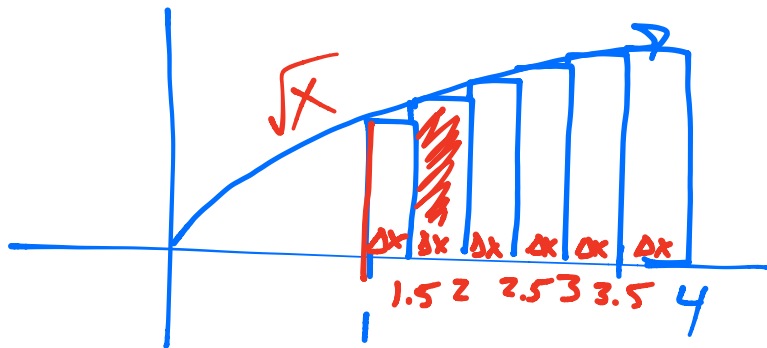
$$\Delta x = \frac{b-a}{n} = \frac{\frac{4\pi}{4} - \frac{\pi}{4}}{2} = \frac{\frac{3\pi}{4} \cdot \frac{1}{2}}{2 \cdot \frac{1}{2}} = \frac{3\pi}{8}$$



$$\begin{aligned} A &= \int_{\frac{\pi}{4}}^{\pi} \sin(x) dx \approx f(x_1) \Delta x + f(x_2) \Delta x \\ &\approx f\left(\frac{\pi}{4}\right) \cdot \frac{3\pi}{8} + f\left(\frac{5\pi}{8}\right) \cdot \frac{3\pi}{8} \\ &\approx \sin\left(\frac{\pi}{4}\right) \cdot \frac{3\pi}{8} + \sin\left(\frac{5\pi}{8}\right) \cdot \frac{3\pi}{8} \\ &\approx \frac{\sqrt{2}}{2} \cdot \frac{3\pi}{8} + (0.9239) \frac{3\pi}{8} \\ &\approx 1.92 \text{ un}^2 \end{aligned}$$



#8) $f(x) = \sqrt{x}$, $a = 1$, $b = 4$, $n = 6$



$$\Delta x = \frac{b-a}{n} = \frac{(4)-(1)}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\begin{aligned} A &= \int_1^4 \sqrt{x} dx \approx f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x + f(x_5) \Delta x + f(x_6) \Delta x \\ &\approx f(1) \cdot \frac{1}{2} + f(1.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2} + f(2.5) \cdot \frac{1}{2} + f(3) \cdot \frac{1}{2} + f(3.5) \cdot \frac{1}{2} \\ &\approx \sqrt{1} \cdot \frac{1}{2} + \sqrt{1.5} \cdot \frac{1}{2} + \sqrt{2} \cdot \frac{1}{2} + \sqrt{2.5} \cdot \frac{1}{2} + \sqrt{3} \cdot \frac{1}{2} + \sqrt{3.5} \cdot \frac{1}{2} \\ &\approx \frac{1}{2} (1 + \sqrt{1.5} + \sqrt{2} + \sqrt{2.5} + \sqrt{3} + \sqrt{3.5}) \end{aligned}$$

$$A \approx 4.41 \text{ un}^2$$

Basic Integration
7.4A – Definite Integrals & Riemann Sum

#9) $f(x) = 3^x$, $a = -1$, $b = 1$, $n = 4$

#10) $f(x) = \frac{2}{x}$, $a = 1$, $b = 2$, $n = 5$

Basic Integration

7.4A – Definite Integrals & Riemann Sum

C: Use the graphing calculator program Riemann to approximate the area using 10, 100, and 1000 rectangles. Round to two decimal places.

#11) $f(x) = 5x, a = 1, b = 2$

#12) $f(x) = x^2 + 1, a = 0, b = 1$

#13) $f(x) = 2 \cos(x), a = 0, b = \frac{\pi}{2}$

#14) $f(x) = 3^x, a = -1, b = 1$

#15) $f(x) = \frac{2}{x}, a = 1, b = 2$

D: Use your graphing calculator to find the area under the curve. Use $fnInt$ from the home screen and do an integral calculation from the graphing utility.

#16) $f(x) = 12 - 3x^2, [1, 2]$

#17) $f(x) = x^{-2}, [1, 4]$

#18) $f(x) = x^{-1/3}, [9, 27]$

#19) $f(x) = 3 \sin(x), [0, \pi]$

#20) $f(x) = e^x, [0, 5]$

Answers

#1) 2.75 un^2

#2) 2.22 un^2

#3) 1.34 un^2

#4) 1.81 un^2

#5) 0.76 un^2

#6) 7 un^2

#7) 1.92 un^2

#8) 4.41 un^2

#9) 1.82 un^2

#10) 1.49 un^2

#11) $7.25 \text{ un}^2, 7.48 \text{ un}^2, 7.50 \text{ un}^2$

#12) $1.29 \text{ un}^2, 1.33 \text{ un}^2, 1.33 \text{ un}^2$

#13) $2.15 \text{ un}^2, 2.02 \text{ un}^2, 2.00 \text{ un}^2$

#14) $2.17 \text{ un}^2, 2.40 \text{ un}^2, 2.42 \text{ un}^2$

#15) $1.44 \text{ un}^2, 1.39 \text{ un}^2, 1.39 \text{ un}^2$

#16) 5 un^2

#17) $\frac{3}{4} \text{ un}^2$

#18) 7.01 un^2

#19) 6 un^2

#20) 147.41 un^2