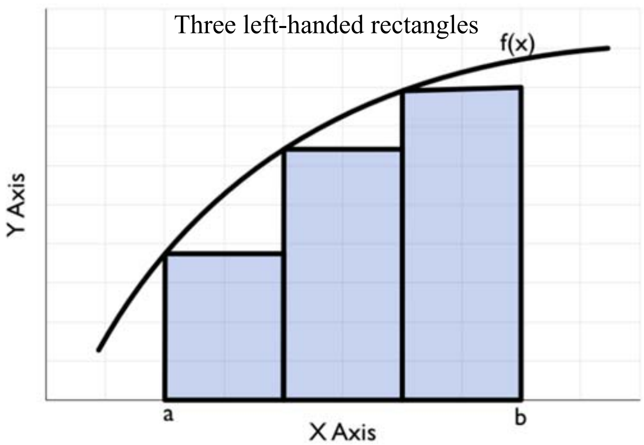
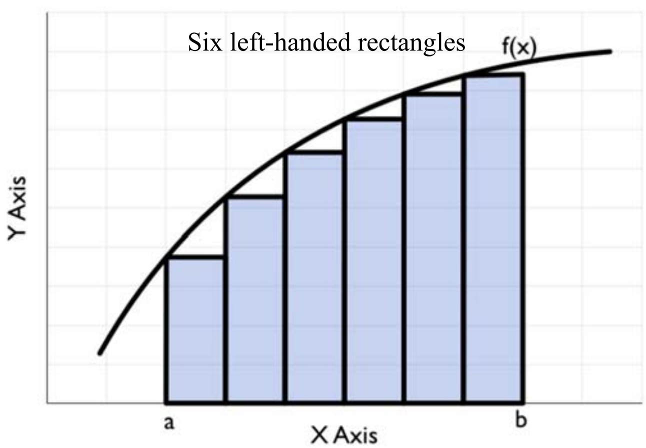
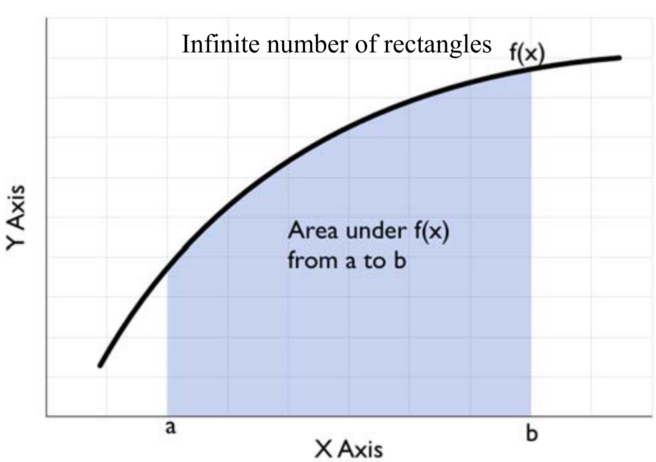
Basic Integration
7.4 – Definite Integrals & Riemann Sum

This section will illustrate how to calculate the area under curves. This is called a *definite integral* of a function.







Definite Integral

Area under *f* from a to b Approximated by *n* Left-Rectangles
Approximating the area under a nonnegative function f by n rectangles…

1. Calculate the rectangle width $\Delta x = \dfrac{b-a}{n}$.

2. Find x-values $x_1, x_2, \ldots, x_n$ by successive additions of Δx beginning with $x_1 = a$.

3. Calculate the sum: $f(x_1) \bullet \Delta x + f(x_2) \bullet \Delta x + f(x_3) \bullet \Delta x + \ldots + f(x_n) \bullet \Delta x$

The Riemann Sum

The sum in step 3 is called the Riemann sum. The *limit* of the Riemann sum as the number *n* approaches infinity gives the *area under the curve* and is called *the definite integral of the function f from a to b,*

written $\int_a^b f(x)dx$.

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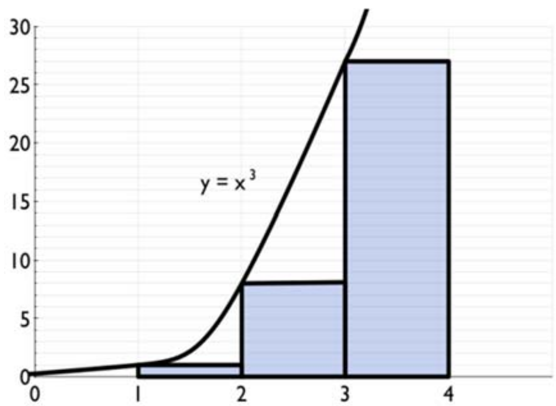
Ex A: Finding area given a graph

#1) Find the sum of the areas of the shaded rectangles. Round answers to two decimal places.



$$\Delta x = \frac{b-a}{n} = \frac{(4)-(1)}{3} = \frac{3}{3} = 1$$

$$A = \int_1^4 x^3\,dx \approx f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$

$$\approx f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$$

$$\approx (1)^3\cdot 1 + (2)^3\cdot 1 + (3)^3\cdot 1$$

$$\approx 1\cdot 1 + 8\cdot 1 + 27\cdot 1$$

$$\approx 1 + 8 + 28$$

$$A \approx 37 \text{ un}^2$$

Ex B: Approximating Area by Rectangles.

#1) Approximate the area under $f(x) = x^2$ from 1 to 2 by five rectangles. Use rectangles with equal bases and with heights equal to the height of the curve at the left-hand edge of the rectangles.

$$\boxed{\Delta x = \frac{b-a}{n} = \frac{(2)-(1)}{5} = \frac{1}{5}}$$

$$A = \int_1^2 x^2\,dx \approx f(x_1)\cdot\Delta x + f(x_2)\cdot\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$$

$$\approx f(1)\cdot\tfrac{1}{5} + f\left(\tfrac{6}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{7}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{8}{5}\right)\cdot\tfrac{1}{5} + f\left(\tfrac{9}{5}\right)\cdot\tfrac{1}{5}$$

$$\approx (1)^2\cdot\tfrac{1}{5} + \left(\tfrac{6}{5}\right)^2\cdot\tfrac{1}{5} + \left(\tfrac{7}{5}\right)^2\cdot\tfrac{1}{5} + \left(\tfrac{8}{5}\right)^2\cdot\tfrac{1}{5} + \left(\tfrac{9}{5}\right)^2\cdot\tfrac{1}{5}$$

$$\approx 1\cdot\tfrac{1}{5} + \tfrac{36}{25}\cdot\tfrac{1}{5} + \tfrac{49}{25}\,\tfrac{1}{5} + \tfrac{64}{25}\,\tfrac{1}{5} + \tfrac{81}{25}\cdot\tfrac{1}{5}$$

$$\approx \tfrac{25}{125} + \tfrac{36}{125} + \tfrac{49}{125} + \tfrac{64}{125} + \tfrac{81}{125}$$

$$\approx \tfrac{255}{125}$$

$$A \approx 2.04\ \text{un}^2$$

Pro Tips

1. Calculate the rectangle widths $\Delta x = \frac{b-a}{n}$

2. Find x-values x_1, $x_2, \ldots, x_n$ by successive additions of Δx beginning with $x_1 = a$.

3. Calculate the sum: $f(x_1) \bullet \Delta x + f(x_2) \bullet \Delta x + f(x_3) \bullet \Delta x + \ldots + f(x_n) \bullet \Delta x$

C: Use the graphing calculator program Riemann to approximate the area using 10, 100, and 1000 rectangles. Round to two decimal places.

#1) $f(x) = 5x + 3$, a = 2, b = 20

If $n = 10$, $A \approx 963$ un^2

If $n = 100$, $A \approx 1035.9$ un^2

If $n = 1000$, $A \approx 1043.19$ un^2

#2) $f(x) = x^2 + 2$, a = 2, b = 10

If $n = 10$, $A \approx 309.12$

If $n = 100$, $A \approx 342.8352$

If $n = 1000$, $A \approx 346.282752$

D: Use your graphing calculator to find the area under the curve. Use *fnInt* from the home screen and do an integral calculation from the graphing utility.

#1) $f(x) = 25 - x^2$, [-5, 5]

fnInt$(25 - x^2, x, -5, 5) = 166.\overline{6}$ un^2

#2) $f(x) = x^3 + 1$, [0, 4]

fnInt$(x^3 + 1, x, 0, 4) = 68$ un^2

Review

Trig Review

$\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

#1) Find $\sin\left(\frac{11\pi}{6}\right)$

#2) Find $\sec\left(\frac{2\pi}{3}\right)$

#3) Find $\cot\left(\frac{4\pi}{3}\right)$

#4) Find $\cos(7\pi)$