# Basic Integration
## Review Chapter 7

Find each integral.

#1) $\int (12x^3 + 3x^2 - 5)dx$

$= 3x^4 + x^3 - 5x + C$

#2) $\int 9\cos(x)\tan(x)\,dx$

$= 9\int \cancel{\cos(x)} \cdot \frac{\sin(x)}{\cancel{\cos(x)}}\,dx$

$= 9\int \sin(x)\,dx$

$= -9\cos(x) + C$

#3) $\int \frac{x^2 - 1}{x - 1}dx$

$= \int \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\,dx$

$= \int (x+1)\,dx$

$= \frac{1}{2}x^2 + x + C$

#4) $\int \left[\frac{\sec(x)}{\cos(x)} - \frac{\tan(x)}{\cot(x)}\right]dx$

$= \int \left(\frac{\sec(x)}{\frac{1}{\sec(x)}} - \frac{\tan(x)}{\frac{1}{\tan(x)}}\right)dx$

$= \int \left[\sec^2(x) - \tan^2(x)\right]dx$

$= \int \left[\tan^2(x) + 1 - \tan^2(x)\right]dx$

$= \int 1\,dx$

$= x + C$

#5) $\int \sqrt{(\csc(x) - 1)(\csc(x) + 1)}\,dx$

$= \int \sqrt{\csc^2(x) - 1}\,dx$

$= \int \sqrt{\cot^2(x)}\,dx$

$= \int \cot(x)\,dx$

$= \ln|\sin(x)| + C$

#6) $\int (x^2 + x + 1 + x^{-1} + x^{-2})dx$

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x| - x^{-1} + C$

#7) $\int (6e^{\frac{2x}{3}})dx$

$= 6\left(\frac{3}{2}\right)e^{\frac{2x}{3}} + C$

$= 9e^{\frac{2x}{3}} + C$

#8) $\int (e^{3x} - \frac{3}{x})dx$

$= \frac{1}{3}e^{3x} - 3\ln|x| + C$

#9) A company's marginal cost function is $MC(x) = 21x^{4/3} - 6x^{\frac{1}{2}} + 50$, where x is the number of units, and fixed costs are \$3000. Find the cost function.

$x = \#$ of units
$C(x) =$ Total Cost

$C(x) = \int (21x^{4/3} - 6x^{\frac{1}{2}} + 50)\,dx$

$= 21\left(\frac{3}{7}\right)x^{7/3} - 6\left(\frac{2}{3}\right)x^{3/2} + 50x + C$

$C(x) = 9x^{7/3} - 4x^{3/2} + 50x + C$

$3000 = 9(0)^{7/3} - 4(0)^{3/2} + 50(0) + C$

$3000 = C$

$C(x) = 9x^{7/3} - 4x^{3/2} + 50x + 3000$

#10) A factory installs new equipment that is expected to generate savings at the rate of $800e^{-0.2t}$ dollars per year, where t is the number of years that the equipment has been in operation.

a. Find a formula for the total savings that the equipment will generate during its first t years.
b. If the equipment originally cost \$2000, when will it "pay for itself"?

(0,0)

T = total \$
t = years

a. $T = \int 800e^{-0.2t}\,dt$

$= 800(-5)e^{-0.2t} + C$

$T = -4000e^{-0.2t} + C$

$0 = -4000e^{-0.2(0)} + C$

$0 = -4000e^{0} + C$

$0 = -4000(1) + C$

$4000 = C$

$T = -4000e^{-0.2t} + 4000$

b. $2000 = -4000e^{-0.2t} + 4000$

$-2000 = -4000e^{-0.2t}$

$\frac{1}{2} = e^{-0.2t}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.2t}\right)$

$\ln\left(\frac{1}{2}\right) = -0.2t$

$-5\ln\left(\frac{1}{2}\right) = t$

$3.5 \approx t$

The equipment will pay for itself in 3.5 years

#11) A flu epidemic hits a college community, beginning with five cases on day t = 0. The rate of growth of the epidemic (new cases per day) is given by $r(t) = 18e^{0.05t}$, where t is the number of days since the epidemic began.

a. Find a formula for the total number of cases of flue in the first t days.
b. Use your answer to part (a) to find the total number of cases in the first 20 days.

(0,5)

F = total flu cases
t = days

a. $F = \int 18e^{0.05t}\,dt$

$= 18(20)e^{0.05t} + C$

$F = 360e^{0.05t} + C$

$5 = 360e^{0.05(0)} + C$

$5 = 360e^{0} + C$

$5 = 360(1) + C$

$-355 = C$

$F = 360e^{0.05t} - 355$

b. $F(20) = 360e^{0.05(20)} - 355$

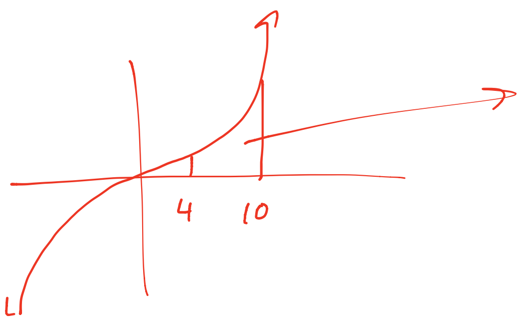
$= 360e^{1} - 355$

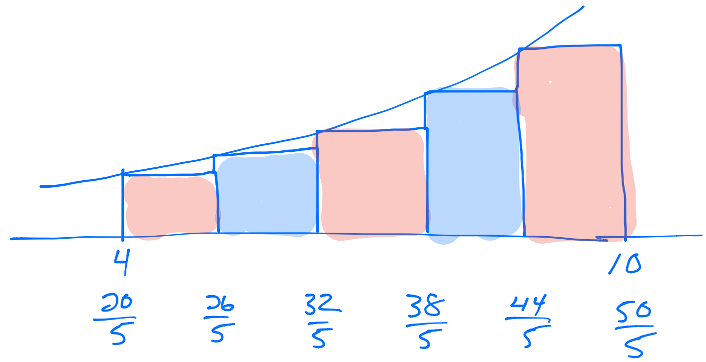
$F(20) \approx 624$ cases

There will be 624 flu cases during the first 20 days.

Find the area under the curve using 5 rectangles. Draw a sketch of the area

#12) $f(x) = x^3$ from x = 4 to x = 10




$$\Delta x = \frac{b-a}{n} = \frac{10-4}{5} = \frac{6}{5}$$

$$A = \int_4^{10} x^3\,dx$$

$$\approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\cdot\Delta x + f(x_5)\cdot\Delta x$$

$$\approx f(4)\cdot\frac{6}{5} + f\left(\frac{26}{5}\right)\cdot\frac{6}{5} + f\left(\frac{32}{5}\right)\cdot\frac{6}{5} + f\left(\frac{38}{5}\right)\frac{6}{5} + f\left(\frac{44}{5}\right)\cdot\frac{6}{5}$$

$$\approx \frac{6}{5}\left[(4)^3 + \left(\frac{26}{5}\right)^3 + \left(\frac{32}{5}\right)^3 + \left(\frac{38}{5}\right)^3 + \left(\frac{44}{5}\right)^3\right]$$

$$\approx \frac{6}{5}\left[\cancel{64}\,\tfrac{8000}{125} + \frac{17576}{125} + \frac{32768}{125} + \frac{54872}{125} + \frac{85184}{125}\right]$$

$$\approx \frac{6}{5}\left[\frac{198400}{125}\right]$$

$$\approx \frac{1{,}190{,}400}{625}$$

$$\boxed{A \approx 1904.64 \text{ un}^2}$$

#13) Explain how a Riemann Sum is used to calculate integrals.

#1: $\int (12x^3 + 3x^2 - 5)dx = 3x^4 + x^3 - 5x + C$

#2: $\int 9\cos(x)\tan(x)\,dx = -9\cos(x) + C$

#3: $\int \frac{x^2 - 1}{x - 1}dx = \frac{1}{2}x^2 + x + C$

#4: $\int \left[\frac{\sec(x)}{\cos(x)} - \frac{\tan(x)}{\cot(x)}\right]dx = x + C$

#5: $\int \sqrt{(\csc(x) - 1)(\csc(x) + 1)}\,dx =$
$\ln[\sin(x)] + C$

#6: $\int (x^2 + x + 1 + x^{-1} + x^{-2})dx =$
$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x| - x^{-1} + C$

#7: $\int (6e^{\frac{2x}{3}})dx = 9e^{\frac{2}{3}x} + C$

#8: $\int (e^{3x} - \frac{3}{x})dx = \frac{1}{3}e^{3x} - 3\ln|x| + C$

#9: $C(x) = 9x^{\frac{7}{3}} - 4x^{\frac{3}{2}} + 50x + 3000$

#10: a.) $f(t) = -4000e^{-0.2t} + 4000$
b.) In about 3.47 years, the equipment will have paid for itself.

#11: a.) $f(t) = 360e^{0.05t} - 355$

b.) After 20 days, the total number of cases of the flu epidemic is about 624.

#12: Area = $\int_4^{10} x^3 dx \approx 1904.64 un.^2$

#13: The Riemann Sum uses rectangles to estimate the area under curves. The width of the rectangles is found by (b – a)/n. The height of each rectangle is found by evaluating the function at each subinterval on the x-axis (the left side of the rectangle). Finally each height is multiplied by the width, and then they are added together. The Riemann Sum method is only an estimation of the area; it does not give an *exact* answer. However, the more rectangles you use, the more accurate your answer is.