Basic Integration
8.1 – Fundamental Theorem of Integral Calculus

Fundamental Theorem of Integral Calculus

$$\int_a^b f(x)dx = F\Big|_a^b = F(b) - F(a)$$

for a continuous function f on an interval $[a, b]$, where F is any antiderivative of f.

Steps to Evaluate a Definite integral

#1: Find an *indefinite* integral of the function (omitting the + C)

#2: *Evaluate* the result at b and *subtract* the evaluation at a.

Properties of Definite Integrals

$$\int_a^b c \cdot f(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

Total Accumulation at a Given Rate:

$$Total = \int (rate)dx$$

A function followed by a vertical bar $\Big|_a^b$ with numbers a and b means evaluate the function at the *upper* number b and then subtract the evaluation at the *lower* number a.

Ex. A: Using the Evaluation Notation

#1) $x^2\Big|_3^5 = (5)^2 - (3)^2$

$= 25 - 9$

$= 16$

#2) $\sqrt{x}\Big|_4^9 = \sqrt{(9)} - \sqrt{(4)}$

$= 3 - 2$

$= 1$

Ex. B: Finding a Definite Integral by the Fundamental Theorem

#1) $\int_1^2 x^2 dx = \frac{1}{3}x^3\Big|_1^2$

$= \frac{1}{3}(2)^3 - \frac{1}{3}(1)^3$

$= \frac{1}{3}(8) - \frac{1}{3}(1)$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$

#2) $\int_0^2 4x^3 dx = x^4\Big|_0^2$

$= (2)^4 - (0)^4$

$= 16 - 0$

$= 16$

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Ex. C: Finding Area Using Fundamental Theorem

#1) Find the area under $y = e^{3x}$ from $x = 0$ to $x = 1$.

Sentence Answer:

$$A = \int_0^1 e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_0^1$$
$$= \frac{1}{3}e^{3(1)} - \frac{1}{3}e^{3(0)}$$
$$= \frac{1}{3}e^{3} - \frac{1}{3}e^{0}$$
$$= \left(\frac{1}{3}e^{3} - \frac{1}{3}\right) un^2$$

Leave the answer in its "exact" form (in terms of the number *e*). In a word problem we would use a calculator to approximate this answer.

#2) Find the area under $f(x) = x^{-1}$ from $x = 1$ to $x = e$.

Sentence Answer:

$$A = \int_1^e x^{-1}\,dx = \ln|x|\Big|_1^e$$
$$= \ln|e| - \ln|1|$$
$$= 1 - 0$$
$$A = 1\ un^2$$

#3) Find the area under $y = 16 - 9x^2$ from –1 to 1.

Sentence Answer:

$$A = \int_{-1}^{1} (16 - 9x^2)\,dx = (16x - 3x^3)\Big|_{-1}^{1}$$
$$= [16(1) - 3(1)^3] - [16(-1) - 3(-1)^3]$$
$$= [16 - 3(1)] - [-16 - 3(-1)]$$
$$= [16 - 3] - [-16 + 3]$$
$$= [13] - [-13]$$
$$A = 26\ un^2$$

Ex. D: Finding the total

T&F Bombs

#1) While indulging his senses at Medieval Times, George is inspired to start yet another business. He conjectures his marginal cost function would be $MC(x) = \frac{50}{\sqrt{x}}$ where x is the number of tar and feather bombs produced. Find the total cost of making tar and feather bombs 100 to 400.

1) Integrate the Marginal Cost Function.

2) Evaluate it from 400 to 100

3) Subtract these values.

$$C = \int_{100}^{400} 50x^{-\frac{1}{2}}\,dx$$
$$= 100x^{\frac{1}{2}}\Big|_{100}^{400}$$
$$= 100\sqrt{(400)} - 100\sqrt{100}$$
$$= 100(20) - 100(10)$$
$$= 2000 - 1000$$
$$C = 1000$$

The total cost of making T+F Bombs 100 to 400 is $ 1000

Poison Arrow Tips

#2) Trying to diversify, George decides to also manufacture poison arrow tips. Being hip to his own mortality, George enlists the aid of some local teen runaways as QATs, quality assurance technicians. A QAT can test poison arrow tips at the ~~rate of $-3t^2 + 15t + 8$ tips~~ per second (for $t \le 6$ because 6 seconds after licking the first tip, poison sets in and kills the technician), where t is the number of seconds after 9:00 A.M. How many tips can be tested between 9:00 and 1 second A.M. and 9:00 and 4 seconds A.M.?

The total work accomplished is the integral of this rate from t =1 (9 AM and 1 second) to t=4 (9 AM and 4 seconds)

$$T = \int_1^4 (-3t^2 + 15t + 8)\,dt$$
$$= \left(-t^3 + \frac{15}{2}t^2 + 8t\right)\Big|_1^4$$
$$= \left[-(4)^3 + \frac{15}{2}(4)^2 + 8(4)\right] - \left[-(1)^3 + \frac{15}{2}(1)^2 + 8(1)\right]$$
$$= \left[-64 + \frac{15}{2}(16) + 32\right] - \left[-1 + \frac{15}{2} + 8\right]$$
$$= [-32 + 120] - [7 + 7.5]$$
$$= [88] - [14.5]$$
$$T = 73.5 \text{ tips}$$

Seventy three and a half arrow tips can be tested from 9:00 and 1 second to 9:00 and 4 seconds.