Basic Integration
8.2A – Average Value

A: Find the average value of each function over the given interval.

#1) $f(x) = x^2, [0, 2]$

$$AV = \frac{1}{2-0}\int_0^2 x^2\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{3}x^3\Big|_0^2$$

$$= \frac{1}{6}x^3\Big|_0^2$$

$$= \left[\frac{1}{6}(2)^3\right] - \left[\frac{1}{6}(0)^3\right]$$

$$= \frac{8}{6} - 0$$

$$AV = \frac{4}{3}$$

#2) $f(x) = \sin(x),\ [0, 2\pi]$ *(also explain why the answer makes sense graphically)*

$$AV = \frac{1}{2\pi-0}\int_0^{2\pi} \sin(x)\,dx$$

$$= \frac{1}{2\pi}\left(-\cos(x)\right)\Big|_0^{2\pi}$$

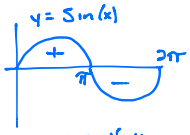
$$= -\frac{1}{2\pi}\cos(x)\Big|_0^{2\pi}$$

$$= \left[-\frac{1}{2\pi}\cos(2\pi)\right] - \left[-\frac{1}{2\pi}\cos(0)\right]$$

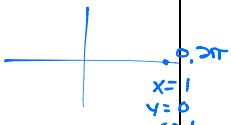
$$= \left[-\frac{1}{2\pi}(1)\right] - \left[-\frac{1}{2\pi}(1)\right]$$

$$= -\frac{1}{2\pi} + \frac{1}{2\pi}$$

$$AV = 0$$



Exactly half the graph is above the x-axis & half the area is below. Total Area = 0.



$\cos(0) = \cos(2\pi) = \frac{1}{1} = 1$

#3) $f(x) = x^{1/3}, [0,10]$

$$AV = \frac{1}{10-0}\int_0^{10} x^{\frac{1}{3}}\,dx$$

$$= \frac{1}{10}\left(\frac{3}{4}\right)x^{4/3}\Big|_0^{10}$$

$$= \frac{3}{40}\sqrt[3]{x^4}\Big|_0^{10}$$

$$= \left[\frac{3}{40}\sqrt[3]{(10)^4}\right] - \left[\frac{3}{40}\sqrt[3]{(0)^4}\right]$$

$$= \frac{3}{40}\sqrt[3]{10000} - 0$$

$$AV \approx 1.62$$

#4) $f(x) = \frac{1}{x^2}, [2, 7]$

$$AV = \frac{1}{7-2}\int_2^7 x^{-2}\,dx$$

$$= \frac{1}{5}(-1)x^{-1}\Big|_2^7$$

$$= \frac{-1}{5x}\Big|_2^7$$

$$= \left[\frac{-1}{5(7)}\right] - \left[\frac{-1}{5(2)}\right]$$

$$= \frac{-1\cdot 2}{5(7)\cdot 2} + \frac{1\cdot 7}{5(2)\cdot 7}$$ (Common Denom)

$$= \frac{-2}{70} + \frac{7}{70}$$

$$= \frac{5}{70}$$

$$AV = \frac{1}{14}$$

#5) $f(x) = \cos(x), \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ (give exact answer)

$$AV = \frac{1}{\frac{5\pi}{2} - \frac{3\pi}{2}}\int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \cos(x)\,dx$$

$$= \frac{1}{\frac{2\pi}{2}}\sin(x)\Big|_{\frac{3\pi}{2}}^{\frac{5\pi}{2}}$$

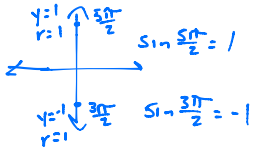
$$= \frac{1}{\pi}\sin(x)\Big|_{\frac{3\pi}{2}}^{\frac{5\pi}{2}}$$

$$= \left[\frac{1}{\pi}\sin\left(\frac{5\pi}{2}\right)\right] - \left[\frac{1}{\pi}\sin\left(\frac{3\pi}{2}\right)\right]$$

$$= \left[\frac{1}{\pi}(1)\right] - \left[\frac{1}{\pi}(-1)\right]$$

$$= \frac{1}{\pi} + \frac{1}{\pi}$$

$$AV = \frac{2}{\pi}$$



#6) $f(x) = 7, [10,100]$

$$AV = \frac{1}{100-10}\int_{10}^{100} 7\,dx$$

$$= \frac{1}{90}7x\Big|_{10}^{100}$$

$$= \frac{7(100)}{90} - \frac{7(10)}{90}$$

$$= \frac{70}{9} - \frac{7}{9}$$

$$= \frac{63}{9}$$

$$AV = 7$$

#7) $f(x) = e^{5x}, [1,3]$

$$AV = \frac{1}{3-1}\int_1^3 e^{5x}\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{5}e^{5x}\Big|_1^3$$

$$= \frac{1}{10}e^{5x}\Big|_1^3$$

$$= \left[\frac{1}{10}e^{5(3)}\right] - \left[\frac{1}{10}e^{5(1)}\right]$$

$$AV = \left(\frac{1}{10}e^{15} - \frac{1}{10}e^{5}\right)$$

#8) $f(x) = 12x^{-1}, [5,11]$

$$AV = \frac{1}{11-5}\int_5^{11} 12x^{-1}\,dx$$

$$= \frac{1}{6}12\ln|x|\Big|_5^{11}$$

$$= [2\ln|11|] - [2\ln|5|]$$

$$= 2[\ln 11 - \ln 5]$$

$$AV = 2\ln\frac{11}{5}$$

#9) $f(x) = e^{0.01t}, [0,5]$

$$AV = \frac{1}{5-0}\int_0^5 e^{0.01t}\,dt$$

$$= \frac{1}{5}(100)e^{0.01t}\Big|_0^5$$

$$= 20e^{0.01t}\Big|_0^5$$

$$= [20e^{0.01(5)}] - [20e^{0.01(0)}]$$

$$= 20e^{0.05} - 20e^{0}$$

$$= 20e^{0.05} - 20$$

#10) $f(x) = \frac{6}{x}, [e, e^2]$

$$AV = \frac{1}{e^2-e}\int_e^{e^2}\frac{6}{x}\,dx$$

$$= \frac{1}{e^2-e}6\ln|x|\Big|_e^{e^2}$$

$$= \left[\frac{6}{e^2-e}\ln|e^2|\right] - \left[\frac{6}{e^2-e}\ln|e|\right]$$

$$= \frac{6}{e^2-e}2 - \frac{6}{e^2-e}$$

$$= \frac{12}{e^2-e} - \frac{6}{e^2-e}$$

$$AV = \frac{6}{e^2-e}$$

Shark Cages

#11) Never missing an opportunity to make some cash, George decides to make shark cages out of the deer carcasses he finds along the highway. George's Discount Shark Cages' sales on day x are given by the function $S(x) = 400x + 6x^2$. Find the average sales during the first 10 days (day 0 to 10).

AS = Average Sale of cages
x = day of Sale

$$AS = \frac{1}{10-0}\int_0^{10}(400x+6x^2)\,dx$$

$$= \frac{1}{10}(200x^2+2x^3)\Big|_0^{10}$$

$$= (20x^2 + \tfrac{1}{5}x^3)\Big|_0^{10}$$

$$= \left[20(10)^2 + \tfrac{1}{5}(10)^3\right] - \left[20(0)^2 + \tfrac{1}{5}(0)^3\right]$$

$$= [20(100) + \tfrac{1}{5}(1000)] - [0]$$

$$= 2000 + 200$$

$$AS = 2200$$

The average sales of shark cages for the first 10 days is $2200.

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George's Jose's Bear

#12) George has an imaginary cat named José. After practicing for x days, José the cat can build a bear suit in $m(x) = \frac{10}{\sqrt{x}}$ minutes. Find the average time to build a bear suit from end of session 1 to the end of session 7.

x = days of practice
AM = Average minutes to build suit

$$AM = \frac{1}{7-1}\int_1^7 10x^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{6}\,20\,x^{\frac{1}{2}}\Big|_1^7$$

$$= \left[\frac{1}{6}\,20\sqrt{7}\right] - \left[\frac{1}{6}\,20\sqrt{1}\right]$$

$$= \frac{1}{6}\,20\sqrt{7} - \frac{1}{6}\,20$$

$$= \frac{20\sqrt{7}}{6} - \frac{20}{6}$$

$$= \frac{20\sqrt{7}-20}{6}$$

$$= \frac{2(10\sqrt{7}-10)}{6}$$

$$= \frac{10\sqrt{7}-10}{3}$$

$AM \approx 5.5$ minutes

At the end of session 1 to end of 7, it will take an average of 5.5 minutes to build bear suit.

America's Most Resilient Colony

#13) Living in George's underwear drawer, an ant colony is $A(x) = 30e^{0.1x}$ ants strong after x months. Find the average size of the ant colony during the first year.

x = months
AS = Average size of ANT Colony

$$AS = \frac{1}{12-0}\int_0^{12} 30e^{0.1x}\,dx$$

$$= \frac{1}{12}\,30(10)e^{0.1x}\Big|_0^{12}$$

$$= 25e^{0.1x}\Big|_0^{12}$$

$$= \left[25e^{0.1(12)}\right] - \left[25e^{0.1(0)}\right]$$

$$= 25e^{1.2} - 25e^{0}$$

$$= 25e^{1.2} - 25$$

$AS = 58$

The average size of the ANT colony is 58 during the first year.

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George's Feet

#14) The temperature of George's cold, sweaty feet at time x hours is $T(x) = -x^2 + 2x + 50$ for $0 \leq x \leq 8$. Find the average temperature between time 2 and 7.

x = hours

AT = Average Temp.

$$AT = \frac{1}{7-2}\int_2^7 (-x^2+2x+50)\,dx$$

$$= \frac{1}{5}\left(-\frac{1}{3}x^3 + x^2 + 50x\right)\Big|_2^7$$

$$= \left(-\frac{1}{15}x^3 + \frac{1}{5}x^2 + 10x\right)\Big|_2^7$$

$$= \left[-\frac{1}{15}(7)^3 + \frac{1}{5}(7)^2 + 10(7)\right] - \left[\frac{1}{15}(2)^3 + \frac{1}{5}(2)^2 + 10(2)\right]$$

$$= \left[-\frac{343}{15} + \frac{49}{5} + 70\right] - \left[-\frac{8}{15} + \frac{4}{5} + 20\right]$$

$$= -\frac{343}{15} + \frac{8}{15} + \frac{49}{5} - \frac{4}{5} + 70 - 20$$

$$= -\frac{335}{15} + \frac{45}{5} + 50$$

$$= -\frac{335}{15} + 9 + 50$$

$$= -\frac{335}{15} + 59$$

$$AT \approx 36.7$$

The average temperature of George's feet is 36.7° between hours 2 and 7.

Answers

#1) $\frac{4}{3}$

#2) 0

#3) about 1.62

#4) $\frac{1}{14}$

#5) $\frac{2}{\pi}$

#6) 7

#7) $\frac{1}{10}(e^{15} - e^5)$

#8) $2\ln\frac{11}{5}$

#9) $20e^{0.05} - 20$

#10) $\frac{6}{e^2 - e}$

#11) The average sales were $2200.

#12) The average time to build a bear is about 4.7 minutes

#13) The colony averaged about 58 ants.

#14) The average temperature is about 36.7 degrees.