Basic Integration
8.2 – Average Value

$Av = \frac{1}{b-a} \int_a^b f(x)\,dx$

Introduction
Much like finding the mean (average) of numbers, we can find the average value of a function using integration. To find the average value, simply divide the integral by the total length of the interval.

Average Value of a Function

$$Average\ Value\ of\ f\ on\ [a, b] = \frac{1}{b-a}\int_a^b f(x)dx$$

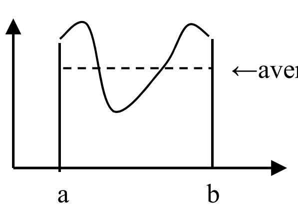
Explanation of Average Value
When integrating a function, you are finding the area under the curve. Any area can be expressed as a rectangle with a length and height. The length is best represented by the interval length.

$$Area_{rectangle} = length \cdot height$$

$$\int_a^b f(x)dx = (b-a)\cdot height$$

$$\frac{1}{b-a}\int_a^b f(x)dx = height$$

$$\frac{1}{b-a}\int_a^b f(x)dx = Average\ Value$$



Ex A: Finding the Average Value of a Function

#1) Find the average value of $f(x) = \frac{2}{3}\sqrt{x}$ from x = 0 to x = 9.

$$Av = \frac{1}{(9)-(0)}\int_0^9 \frac{2}{3}x^{\frac{1}{2}}dx$$

$$= \frac{1}{9}\cdot\frac{2}{3}\left(\frac{2}{3}\right)x^{\frac{3}{2}}\Big|_0^9$$

$$= \frac{4}{81}\left(\sqrt{x}\right)^3\Big|_0^9$$

$$= \frac{4}{81}\left(\sqrt{(9)}\right)^3 - \frac{4}{81}\left(\sqrt{(0)}\right)^3$$

$$= \frac{4}{81}(3)^3 - \frac{4}{81}(0)$$

$$= \frac{4}{81}\cdot 27 - 0$$

$$Av = \frac{4}{3}$$

#2) Find the average value of f(x) = x^2 from x = 0 to x = 2.

$$Av = \frac{1}{2-0}\int_0^2 x^2\,dx$$

$$= \frac{1}{2}\cdot\frac{1}{3}\cdot x^3\Big|_0^2$$

$$= \left[\frac{1}{6}(2)^3\right] - \left[\frac{1}{6}(0)^3\right]$$

$$= \left[\frac{8}{6}\right] - [0]$$

$$Av = \frac{4}{3}$$

Boring Games

#3) The number of horrible, boring video games in the world is predicted to be $P(t) = 263e^{0.01t}$ million games, where t is the number of years since 2005. Find the average number of horrible games between the years 2010 and 2020.

Since t is the number of years since 2005, we will be integrating on the interval [5,15]

$$A = \frac{1}{15-5}\int_5^{15} 263e^{0.01t}\,dt$$

$$= \left(\frac{1}{10}\right) 26300e^{0.01t}\Big|_5^{15}$$

$$= 2630e^{0.01t}\Big|_5^{15}$$

$$= \left[2630e^{0.01(15)}\right] - \left[2630e^{0.01(5)}\right]$$

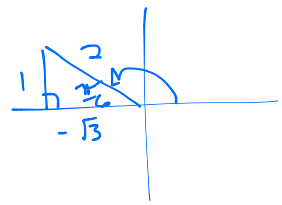
$$= 2630e^{0.15} - 2630e^{0.05}$$

$$\approx 290.781075 \text{ million games}$$

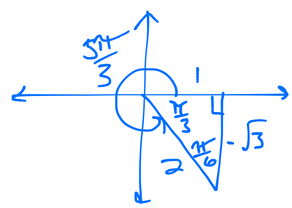
$$A \approx 290{,}781{,}075 \text{ games}$$

Review

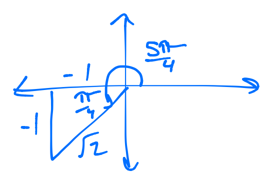
#1) Find $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$



#2) Find $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$



#3) Find $\tan\left(\frac{5\pi}{4}\right) = \frac{-1}{-1} = 1$



#4) Find $\csc(3\pi) = \frac{r}{y} = \frac{1}{0} = \text{und}$