

Basic Integration

9.1A – U Substitution

A: Integrate using a u-substitution.

#1) $\int (x^2 - 1)^5 2x \, dx$

$$= \int u^5 \cdot 2x \left(\frac{du}{2x} \right)$$

$$= \int u^5 \, du$$

$$= \frac{1}{6} u^6 + C$$

$$= \frac{1}{6} (x^2 - 1)^6 + C$$

$$\begin{aligned} u &= x^2 - 1 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

#3) $\int \frac{5x^4}{x^5 - 9} \, dx$

$$= \int \frac{5x^4}{u} \left(\frac{du}{5x^4} \right)$$

$$= \int \frac{1}{u} \, du$$

$$= \ln|u| + C$$

$$= \ln|x^5 - 9| + C$$

$$\begin{aligned} u &= x^5 - 9 \\ \frac{du}{dx} &= 5x^4 \\ du &= 5x^4 \, dx \\ \frac{du}{5x^4} &= dx \end{aligned}$$

#2) $\int e^{x^4} 4x^3 \, dx$

$$= \int e^u \cdot 4x^3 \left(\frac{du}{4x^3} \right)$$

$$= \int e^u \, du$$

$$= e^u + C$$

$$= e^{x^4} + C$$

$$\begin{aligned} u &= x^4 \\ \frac{du}{dx} &= 4x^3 \\ du &= 4x^3 \, dx \\ \frac{du}{4x^3} &= dx \end{aligned}$$

#4) $\int (x^2 - 1)^5 x \, dx$

$$= \int u^5 \cdot x \left(\frac{du}{2x} \right)$$

$$= \frac{1}{2} \int u^5 \, du$$

$$= \frac{1}{2} \cdot \frac{1}{6} u^6 + C$$

$$= \frac{1}{12} (x^2 - 1)^6 + C$$

$$\begin{aligned} u &= x^2 - 1 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

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#5) $\int e^{x^4} 7x^3 dx$

$$\begin{aligned} &= \int e^u 7x^3 \left(\frac{du}{4x^3}\right) \\ &= \frac{7}{4} \int e^u du \\ &= \frac{7}{4} e^u + C \\ &= \frac{7}{4} e^{x^4} + C \end{aligned}$$

$$\begin{aligned} u &= x^4 \\ \frac{du}{dx} &= 4x^3 \\ du &= 4x^3 dx \\ \frac{du}{4x^3} &= dx \end{aligned}$$

B: If possible, integrate by a u-substitution. If not possible, say so.

#7) $\int (x^4 - 9)^3 x^3 dx$

$$\begin{aligned} &= \int u^3 x^3 \left(\frac{du}{4x^3}\right) \\ &= \frac{1}{4} \int u^3 du \\ &= \frac{1}{4} \cdot \frac{1}{4} u^4 + C \\ &= \frac{1}{16} (x^4 - 9)^4 + C \end{aligned}$$

$$\begin{aligned} u &= x^4 - 9 \\ \frac{du}{dx} &= 4x^3 \\ du &= 4x^3 dx \\ \frac{du}{4x^3} &= dx \end{aligned}$$

#6) $\int \frac{x^4}{x^5 - 9} dx$

$$\begin{aligned} &= \int \frac{x^4}{u} \left(\frac{du}{5x^4}\right) \\ &= \frac{1}{5} \int \frac{1}{u} du \\ &= \frac{1}{5} \ln|u| + C \\ &= \frac{1}{5} \ln|x^5 - 9| + C \end{aligned}$$

$$\begin{aligned} u &= x^5 - 9 \\ \frac{du}{dx} &= 5x^4 \\ du &= 5x^4 dx \\ \frac{du}{5x^4} &= dx \end{aligned}$$

#8) $\int (x^4 - 9)^3 x^5 dx$

$$= \int u^3 x^2 x^3 \left(\frac{du}{4x^3}\right)$$

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$$\begin{aligned} u &= x^4 - 9 \\ \frac{du}{dx} &= 4x^3 \\ du &= 4x^3 dx \\ \frac{du}{4x^3} &= dx \end{aligned}$$

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#9) $\int e^{x^3} x^3 dx$

$$= \int e^u x^3 \left(\frac{du}{3x^2}\right)$$

$$= \int e^u x du$$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$\frac{du}{3x^2} = dx$$

CAN'T INTEGRATE
BY SUBSTITUTION

#10) $\int e^{z^2} 3z dz$

$$= \int e^u 3z \left(\frac{du}{2z}\right)$$

$$= \frac{3}{2} \int e^u du$$

$$= \frac{3}{2} e^u + C$$

$$= \frac{3}{2} e^{z^2} + C$$

$$u = z^2$$

$$\frac{du}{dz} = 2z$$

$$du = 2z dz$$

$$\frac{du}{2z} = dz$$

#11) $\int \frac{z^2}{5z^3+1} dz$

$$= \int \frac{z^2}{u} \left(\frac{du}{15z^2}\right)$$

$$= \frac{1}{15} \int \frac{1}{u} du$$

$$= \frac{1}{15} \ln|u| + C$$

$$= \frac{1}{15} \ln|5z^3+1| + C$$

$$u = 5z^3+1$$

$$\frac{du}{dz} = 15z^2$$

$$du = 15z^2 dz$$

$$\frac{du}{15z^2} = dz$$

#12) $\int \frac{dz}{2z+1}$

$$= \int \left(\frac{du}{2}\right) \frac{1}{u}$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|2z+1| + C$$

$$u = 2z+1$$

$$\frac{du}{dz} = 2$$

$$du = 2 dz$$

$$\frac{du}{2} = dz$$

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#13) $\int \sqrt[3]{t^4 + 81} t^3 dt$

$$= \int u^{\frac{1}{3}} \cancel{t^3} \left(\frac{du}{4t^3} \right)$$

$$= \frac{1}{4} \int u^{\frac{1}{3}} du$$

$$= \frac{1}{4} \left(\frac{5}{5} \right) u^{\frac{5}{3}} + C$$

$$= \frac{5}{24} \left(\sqrt[3]{t^4 + 81} \right)^{\frac{5}{3}} + C$$

$$\begin{aligned} u &= t^4 + 81 \\ \frac{du}{dt} &= 4t^3 \\ du &= 4t^3 dt \\ \frac{du}{4t^3} &= dt \end{aligned}$$

#15) $\int (3t^2 + 6t)^3 (6t + 6) dt$

$$= \int u^3 \cancel{(6t+6)} \left(\frac{du}{6t+6} \right)$$

$$= \int u^3 du$$

$$= \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} (3t^2 + 6t)^4 + C$$

$$\begin{aligned} u &= 3t^2 + 6t \\ \frac{du}{dt} &= 6t + 6 \\ du &= (6t + 6) dt \\ \frac{du}{6t+6} &= dt \end{aligned}$$

#14) $\int \sqrt[3]{t^5 - 1} t^3 dt$

$$\begin{aligned} u &= t^5 - 1 \\ \frac{du}{dt} &= 5t^4 \\ du &= 5t^4 dt \end{aligned}$$

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#16) $\int (x^2 + 5x)^3 (4x + 10) dx$

$$= \int u^3 \cancel{2(2x+5)} \left(\frac{du}{2x+5} \right)$$

$$= 2 \int u^3 du$$

$$= 2 \left(\frac{1}{4} \right) u^4 + C$$

$$= \frac{1}{2} (x^2 + 5x)^4 + C$$

$$\begin{aligned} u &= x^2 + 5x \\ \frac{du}{dx} &= 2x + 5 \\ du &= (2x + 5) dx \\ \frac{du}{2x+5} &= dx \end{aligned}$$

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#17) $\int \frac{4x^3+3x^2}{x^4+x^3} dx$

$$= \int \frac{\cancel{4x^3+3x^2}^2 \left(\frac{du}{\cancel{4x^3+3x^2}} \right)}{u}$$

$$= \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln|x^4+x^3| + C$$

$$u = x^4 + x^3$$

$$\frac{du}{dx} = 4x^3 + 3x^2$$

$$du = (4x^3 + 3x^2) dx$$

$$\frac{du}{4x^3 + 3x^2} = dx$$

#19) $\int e^{x^3+x}(6x^2+2) dx$

$$= \int e^u \cancel{(3x^2+1)} \left(\frac{du}{\cancel{3x^2+1}} \right)$$

$$= \int e^u du$$

$$= 2e^u + C$$

$$= 2e^{x^3+x} + C$$

$$u = x^3 + x$$

$$\frac{du}{dx} = 3x^2 + 1$$

$$du = (3x^2 + 1) dx$$

$$\frac{du}{3x^2 + 1} = dx$$

#18) $\int \frac{20x^4+6x}{2x^5+x^2} dx$

$$= \int \frac{2(10x^4+3x)}{u} \left(\frac{du}{10x^4+3x} \right)$$

$$u = 2x^5 + x^2$$

$$\frac{du}{dx} = 10x^4 + 2x$$

$$du = (10x^4 + 2x) dx$$

$$\frac{du}{10x^4 + 2x} = dx$$

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#20) $\int (2x-4)^5 dx$

$$= \int u^5 \left(\frac{du}{2} \right)$$

$$= \frac{1}{2} \int u^5 du$$

$$= \frac{1}{2} \left(\frac{1}{6} u^6 \right) + C$$

$$= \frac{1}{12} (2x-4)^6 + C$$

$$u = 2x - 4$$

$$\frac{du}{dx} = 2$$

$$du = 2 dx$$

$$\frac{du}{2} = dx$$

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#21) $\int \frac{e^x}{e^x-1} dx$

$$= \int \frac{e^x}{u} \left(\frac{dy}{e^x} \right)$$

$$= \int \frac{dy}{u}$$

$$= \ln|u| + C$$

$$= \ln|e^x-1| + C$$

$$\begin{aligned} u &= e^x - 1 \\ \frac{dy}{dx} &= e^x \\ du &= e^x dx \\ \frac{dy}{e^x} &= dx \end{aligned}$$

#23) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ (Pro tip: set $u = \sqrt{x}$)

$$= \int \frac{e^u}{\sqrt{x}} (2\sqrt{x} du)$$

$$= 2 \int e^u du$$

$$= 2e^u + C$$

$$= 2e^{\sqrt{x}} + C$$

$$\begin{aligned} u &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2} x^{-1/2} \\ du &= \frac{1}{2\sqrt{x}} dx \\ 2\sqrt{x} du &= dx \end{aligned}$$

#22) $\int \frac{\ln x}{x} dx$ (Pro tip: set $u = \ln x$)

$$= \int \frac{u}{x} (x du)$$

$$= \int u du$$

$$= \frac{1}{2} u^2 + C$$

$$= \frac{1}{2} (\ln x)^2 + C$$

$$= \frac{1}{2} \ln^2 x + C$$

$$\begin{aligned} u &= \ln x \\ \frac{du}{dx} &= \frac{1}{x} \\ du &= \frac{1}{x} dx \\ x du &= dx \end{aligned}$$

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C: Find each integral by using algebra first.

#24) $\int (x+9)x^3 dx$

$$= \int (x^4 + 9x^3) dx$$

$$= \frac{1}{5}x^5 + \frac{9}{4}x^4 + C$$

#25) $\int (x+3)^2 5x dx$

$$= \int (x^2 + 6x + 9) 5x dx$$

$$= \int (5x^3 + 30x^2 + 45x) dx$$

$$= \frac{5}{4}x^4 + 10x^3 + \frac{45}{2}x^2 + C$$

#26) $\int (x-1)^2 \sqrt{x} dx$

$$= \int (x^2 - 2x + 1) x^{\frac{1}{2}} dx$$

$$= \int (x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 1) x^{\frac{1}{2}} dx$$

$$= \int (x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + x^{\frac{1}{2}}) dx$$

$$= \frac{2}{7}x^{\frac{7}{2}} - 2\left(\frac{2}{5}\right)x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$$

$$= x^{\frac{1}{2}} \left(\frac{2}{7}x^3 - \frac{4}{5}x^{\frac{3}{2}} + \frac{2}{3}x^{\frac{1}{2}} \right) + C$$

$$= \sqrt{x} \left(\frac{2}{7}x^3 - \frac{4}{5}x^2 + \frac{2}{3}x \right) + C$$

#27) $\int (x+4)(x-4) dx$

$$= \int (x^2 - 16) dx$$

$$= \frac{1}{3}x^3 - 16x + C$$

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