Basic Integration
9.2A – Integration Substitution & Trig

A: Integrate the trig functions by substitution when needed

#1) $\int 3\cos(4x)\,dx$

$= \int 3\cos(u)\left(\frac{du}{4}\right)$

$= \frac{3}{4}\int \cos(u)$

$= \frac{3}{4}\sin(u) + C$

$= \frac{3}{4}\sin(4x) + C$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

#2) $\int \sin(4x)\,dx$

$= \int \sin(u)\frac{du}{4}$

$= \frac{1}{4}\int \sin(u)\,du$

$= -\frac{1}{4}\cos(u) + C$

$= -\frac{1}{4}\cos(4x) + C$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

#3) $\int 4\sec(4x)\tan(4x)\,dx$

$= \int 4\sec(u)\tan(u)\left(\frac{du}{4}\right)$

$= \int \sec(u)\tan(u)\,du$

$= \sec(u) + C$

$= \sec(4x) + C$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

#4) $\int \cos(8x)\,dx$

$= \int \cos(u)\left(\frac{du}{8}\right)$

$= \frac{1}{8}\int \cos(u)\,du$

$= \frac{1}{8}\sin(u) + C$

$= \frac{1}{8}\sin(8x) + C$

$u = 8x$

$\frac{du}{dx} = 8$

$du = 8\,dx$

$\frac{du}{8} = dx$

#5) $\int \cos(11x)\sqrt{\sin(11x)}\,dx$

$= \int \cos(11x)\, u^{\frac{1}{2}} \frac{du}{11\cos(11x)}$

$u = \sin(11x)$

$du = 11\cdot\cos(11x)\,dx$

$\frac{du}{11\cos(11x)} = dx$

$= \frac{1}{11}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{11}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{33}\sqrt{\sin^3(11x)} + C$

#6) $\int \sin\left(\frac{\pi}{3}x\right)dx$

$= \int \sin(u)\frac{3}{\pi}du$

$= \frac{3}{\pi}\int \sin(u)\,du$

$u = \frac{\pi}{3}x$

$du = \frac{\pi}{3}dx$

$\frac{3}{\pi}du = dx$

$= \frac{3}{\pi}(-\cos(u)) + C$

$= -\frac{3}{\pi}\cos\left(\frac{\pi}{3}x\right) + C$

#7) $\int \csc(x)\cot(x)\csc^2(x)\,dx$

$= \int \csc(x)\cancel{\cot(x)}\, u\cdot\frac{du}{-\cancel{\cot(x)}}$

$u = \csc^2(x)$

$du = -\cot(x)\csc^2(x)\,dx$

$\frac{du}{-\cot(x)\csc^2(x)} = dx$

$= -\int \csc(x)\cdot u\,du$

$u = \csc^2(x)$

$\sqrt{u} = \csc(x)$

$= -\int \sqrt{u}\,u\,du$

$= -\int u^{3/2}\,du$

$= -\frac{2}{5}u^{5/2} + C$

$= -\frac{2}{5}\left(\sqrt{\csc^2(x)}\right)^5 + C$

$= -\frac{2}{5}\csc^5(x) + C$

#8) $\int \frac{\sec(x)}{\cos(x)}dx$

$= \int \sec^2(x)\,dx$

$= \tan(x) + C$

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#9) $\int 2x\cos(x^2)\,dx$

$= \int 2x\cos(u)\left(\frac{du}{2x}\right)$

$= \int \cos(u)\,du$

$= \sin(u)+C$

$= \sin(x^2)+C$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

#10) $\int \sin(4x)\,dx$

$= \int \sin(u)\left(\frac{du}{4}\right)$

$= \frac{1}{4}\int \sin(u)\,du$

$= -\frac{1}{4}\cos(u)+C$

$= -\frac{1}{4}\cos(4x)+C$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

#11) $\int 3x^2\sec(x^3)\tan(x^3)\,dx$

$= \int 3x^2\sec(u)\tan(u)\left(\frac{du}{3x^2}\right)$

$= \int \sec(u)\tan(u)\,du$

$= \sec(u)+C$

$= \sec(x^3)+C$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

#12) $\int \frac{1}{27}\sin(27x)\,dx$

$= \int \frac{1}{27}\sin(u)\left(\frac{du}{27}\right)$

$= \frac{1}{729}\int \sin(u)\,du$

$= -\frac{1}{729}\cos(u)+C$

$= -\frac{1}{729}\cos(27x)+C$

$u = 27x$

$\frac{du}{dx} = 27$

$du = 27\,dx$

$\frac{du}{27} = dx$

#13) $\int \sec^2(x)\sqrt{\tan(x)}\,dx$

$u = \tan(x)$

$\frac{du}{dx} = \sec^2(x)$

$du = \sec^2(x)\,dx$

$\frac{du}{\sec^2(x)} = dx$

$= \int \sec^2(x)\sqrt{u}\left(\frac{du}{\sec^2(x)}\right)$

$= \int u^{\frac{1}{2}}\,du$

$= \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{\tan^3(x)} + C$

#14) $\int \frac{1}{\pi}\csc^2(\pi x)\,dx$

$u = \pi x$

$\frac{du}{dx} = \pi$

$du = \pi\,dx$

$\frac{du}{\pi} = dx$

$= \int \frac{1}{\pi}\csc^2(u)\left(\frac{du}{\pi}\right)$

$= \frac{1}{\pi^2}\int \csc^2(u)\,du$

$= \frac{1}{\pi^2}\left(-\cot(u)\right) + C$

$= \frac{-1}{\pi^2}\cot(\pi x) + C$

#15) $\int \tan(x)\ln|\sec(x)|\,dx$

$u = \ln|\sec(x)|$

$\frac{du}{dx} = \frac{\sec'(x)}{\sec(x)}$

$\frac{du}{dx} = \frac{\sec(x)\tan(x)}{\sec(x)}$

$\frac{du}{dx} = \tan(x)$

$du = \tan(x)\,dx$

$\frac{du}{\tan(x)} = dx$

$= \int \tan(x)\,u\left(\frac{du}{\tan(x)}\right)$

$= \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\left[\ln|\sec(x)|\right]^2 + C$

#16) $\int \sec(x)\ln|\sec(x)+\tan(x)|\,dx$

$u = \ln|\sec(x)+\tan(x)|$

$\frac{du}{dx} = \sec(x)$

$du = \sec(x)\,dx$

$\frac{du}{\sec(x)} = dx$

$= \int \sec(x)\cdot u\left(\frac{du}{\sec(x)}\right)$

$= \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\left[\ln|\sec(x)+\tan(x)|\right]^2 + C$