Basic Integration
Review Chapter 9

Find each integral by substitution or state that it cannot be evaluated by our substitution formulas.

#1) $\int x^2 \sqrt[3]{x^3 - 1}dx$

$u = x^3 - 1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$= \int \cancel{x^2} \sqrt[3]{u} \frac{du}{3\cancel{x^2}}$

$= \frac{1}{3} \int u^{\frac{1}{3}} du$

$= \frac{1}{3}\left(\frac{3}{4}\right)u^{\frac{4}{3}} + C$

$= \frac{1}{4}\sqrt[3]{(x^3-1)^4} + C$

#2) $\int \frac{dx}{9-3x}$

$u = 9 - 3x$

$\frac{du}{dx} = -3$

$du = -3\,dx$

$\frac{du}{-3} = dx$

$= \int \frac{1}{u} \frac{du}{-3}$

$= -\frac{1}{3} \int \frac{1}{u} du$

$= -\frac{1}{3} \ln|u| + C$

$= -\frac{1}{3} \ln|9-3x| + C$

#3) $\int \frac{e^x}{e^x - 1} dx$

$u = e^x - 1$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$\frac{du}{e^x} = dx$

$= \int \frac{\cancel{e^x}}{u} \frac{du}{\cancel{e^x}}$

$= \int \frac{1}{u} du$

$= \ln|u| + C$

$= \ln|e^x - 1| + C$

#4) $\int x^2 \sqrt{x^4 - 1}dx$

$u = x^4 - 1$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\frac{du}{4x^3}\,dx$

$= \int \cancel{x^2} \sqrt[3]{u} \frac{du}{4\cancel{x^3}}$

$= \frac{1}{4} \int u^{\frac{1}{3}} du$

$= \frac{1}{4}\left(\frac{3}{4}\right)u^{\frac{4}{3}} + C$

$= \frac{3}{16}\sqrt[3]{(x^4-1)^4} + C$

#5) $\int \frac{w+3}{(w^2+6w-1)^2} dw$

$u = w^2 + 6w - 1$

$\frac{du}{dw} = 2w + 6$

$du = (2w+6)dw$

$\frac{du}{2w+6} = dw$

$= \int \frac{\cancel{w+3}}{u^2} \frac{du}{2(\cancel{w+3})}$

$= \frac{1}{2} \int \frac{1}{u^2} du$

$= \frac{1}{2} \int u^{-2} du$

$= \frac{1}{2}(-1)u^{-1} + C$

$= -\frac{1}{2} \frac{1}{w^2+6w-1} + C$

$= \frac{-1}{2(w^2+6w-1)} + C$

Find each definite integral. (A calculator may only be used to check your answer.)

#6) $\int_0^3 x\sqrt{x^2+16}dx$

$u = x^2 + 16$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \int_{x=0}^{x=3} \cancel{x} \sqrt{u} \cdot \frac{du}{2\cancel{x}}$

$= \frac{1}{2} \int_{x=0}^{x=3} u^{\frac{1}{2}} du$

$= \frac{1}{2}\left(\frac{2}{3}\right) u^{\frac{3}{2}} \Big|_{x=0}^{x=3}$

$= \frac{1}{3}\left(\sqrt{x^2+16}\right)^3 \Big|_0^3$

$= \left[\frac{1}{3}\left(\sqrt{(3)^2+16}\right)^3 - \left[\frac{1}{3}\left(\sqrt{(0)^2+16}\right)^3\right]\right.$

$= \frac{1}{3}\left(\sqrt{9+16}\right)^3 - \frac{1}{3}\left(\sqrt{16}\right)^3$

$= \frac{1}{3}\left(\sqrt{25}\right)^3 - \frac{1}{3}\left(\sqrt{16}\right)^3$

$= \frac{1}{3} \cdot 5^3 - \frac{1}{3} \cdot 4^3$

$= \frac{1}{3}(125) - \frac{1}{3}(64)$

$= \frac{125}{3} - \frac{64}{3}$

$= \frac{61}{3}$

#7) $\int_3^9 \frac{dx}{x-2}$

$= \int_{x=3}^{x=9} \frac{1}{u}\,du$

$u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$= \ln|u|\Big|_{x=3}^{x=9}$

$= \ln|x-2|\Big|_3^9$

$= \ln|9-2| - \ln|3-2|$

$= \ln|7| - \ln|1|$

$= \ln 7 - 0$

$= \ln 7$

Find each indefinite integral by substitution.

#8) $\int 20x \sin(5x^2 - 3)\, dx$

$= \int 20\cancel{x} \sin u \frac{du}{10\cancel{x}}$

$u = 5x^2 - 3$

$\frac{du}{dx} = 10x$

$du = 10x\,dx$

$\frac{du}{10x} = dx$

$= 2\int \sin u\, du$

$= -2\cos u + C$

$= -2\cos(5x^2-3) + C$

#9) $\int 6e^{3x} \cos(e^{3x} - 5)\, dx$

$= \int 6\cancel{e^{3x}} \cos u \frac{du}{3\cancel{e^{3x}}}$

$u = e^{3x} - 5$

$\frac{du}{dx} = 3e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{du}{3e^{3x}} = dx$

$= 2\int \cos u\, du$

$= 2\sin u + C$

$= 2\sin(e^{3x}-5) + C$

#10) $\int 16x \sec^2(4x^2 - 2)\, dx$

$= \int 16x \sec^2(u) \frac{du}{8x}$

$u = 4x^2 - 2$

$du = 8x\,dx$

$\frac{du}{8x} = dx$

$= 2\int \sec^2(u)\, du$

$= 2\tan(u) + C$

$= 2\tan(4x^2-2) + C$

#11) $\int \frac{50x}{\sec(5x^2+5)} dx$

$u = 5x^2 + 5$
$du = 10x\, dx$
$\frac{du}{10x} = dx$

$= \int \frac{50x}{\sec(u)} \frac{du}{10x}$

$= 5 \int \frac{1}{\sec(u)} du$

$= 5 \int \cos(u)\, du$

$= 5\sin(u) + C$

$= 5\sin(5x^2+5) + C$

Review Chapter 9

#1: $\frac{1}{4}(x^3-1)^{\frac{4}{3}} + C$

#2: $-\frac{1}{3}\ln|9-3x| + C$

#3: $\ln|e^x - 1| + C$

#4: It cannot be integrated by substitution because the powers of *du* and the integral do not match.

#5: $-\frac{1}{2}(w^2+6w-1)^{-1} + C$

#6: $\frac{61}{3}$

#7: $\ln 7$

#8: $-2\cos(5x^2-3) + C$

#9: $2\sin(e^{3x}-5) + C$

#10: $2\tan(4x^4-2) + C$

#11: $5\sin(5x^2+5) + C$