Basic Integration
Cumulative Review Chapters 7 - 9

Find each integral.

#1) $\int(6\sqrt{x}-5)dx$

$= \int(6x^{\frac{1}{2}}-5)dx$

$= 6(\frac{2}{3})x^{\frac{3}{2}} - 5x + C$

$= 4\sqrt{x^3} - 5x + C$

#2) $\int(5\sqrt{x^3}-6x)dx$

$= \int(5x^{3/2} - 6x)dx$

$= 5(\frac{2}{5})x^{5/2} - 6(\frac{1}{2})x^2 + C$

$= 2\sqrt{x^5} - 3x^2 + C$

#3) $\int(x-4)(x+4)dx$

$= \int(x^2-16)dx$

$= \frac{1}{3}x^3 - 16x + C$

#4) $\int\frac{3x^3+2x^2+4x}{x}dx$

$= \int\frac{x(3x^2+2x+4)}{x}dx$

$= \int(3x^2+2x+4)dx$

$= x^3 + x^2 + 4x + C$

#5) $\int e^{\frac{x}{2}}dx$

$= \int e^{\frac{1}{2}x}dx$

$= 2e^{\frac{1}{2}x} + C$

#6) $\int 4x^{-1}dx$

$= 4\ln|x| + C$

#7) $\int(9x^2+2x^{-1}+6e^{3x})dx$

$= 9(\frac{1}{3})x^3 + 2\ln|x| + 6(\frac{1}{3})e^{3x} + C$

$= 3x^3 + 2\ln|x| + 2e^{3x} + C$

#8) $\int\left(\frac{1}{x^2}+\frac{1}{x}+e^{-x}\right)dx$

$= \int(x^{-2} + \frac{1}{x} + e^{-x})dx$

$= -x^{-1} + \ln|x| - e^{-x} + C$

$= \frac{-1}{x} + \ln|x| - e^{-x} + C$

Find each integral.

#9) $\int_1^9 \left(x - \frac{1}{\sqrt{x}}\right) dx$

$= \int_1^9 (x - x^{-\frac{1}{2}})dx$

$= \left(\frac{1}{2}x^2 - 2x^{\frac{1}{2}}\right)\Big|_1^9$

$= \left(\frac{1}{2}x^2 - 2\sqrt{x}\right)\Big|_1^9$

$= \left[\frac{1}{2}(9)^2 - 2\sqrt{9}\right] - \left[\frac{1}{2}(1)^2 - 2\sqrt{1}\right]$

$= \left[\frac{1}{2}(81) - 2(3)\right] - \left[\frac{1}{2}(1) - 2(1)\right]$

$= \frac{81}{2} - 6 - \frac{1}{2} + 2$

$= \frac{80}{2} - 4$

$= 40 - 4$

$= 36$

#10) $\int_1^{e^4} \frac{dx}{x}$

$= \ln|x| \Big|_1^{e^4}$

$= \ln|e^4| - \ln|1|$

$= \ln e^4 - 0$

$= \ln e^4$

$= 4 \ln e$

$= 4$

#11) $\int_0^{100} (e^{0.05x} - e^{0.001x}) dx$

$= \left[20e^{0.05x} - 1000e^{0.001x}\right]\Big|_0^{100}$

$= \left[20e^{0.05(100)} - 1000e^{0.001(100)}\right] - \left[20e^{0.05(0)} - 1000e^{0.001(0)}\right]$

$= \left[20e^5 - 1000e^{0.1}\right] - \left[20e^0 - 1000e^0\right]$

$= \left[20e^5 - 1000e^{0.1}\right] - \left[20(1) - 1000(1)\right]$

$= \left[20e^5 - 1000e^{0.1}\right] - \left[20 - 1000\right]$

$= \left[20e^5 - 1000e^{0.1}\right] - \left[-980\right]$

$= 20e^5 - 1000e^{0.1} + 980$

#12) $\int_2^5 (3x^2 - 4x + 5) dx$

$= \left[x^3 - 2x^2 + 5x\right]\Big|_2^5$

$= \left[(5)^3 - 2(5)^2 + 5(5)\right] - \left[(2)^3 - 2(2)^2 + 5(2)\right]$

$= \left[125 - 2(25) + 25\right] - \left[8 - 2(4) + 10\right]$

$= \left[125 - 50 + 25\right] - \left[8 - 8 + 10\right]$

$= \left[125 - 25\right] - \left[10\right]$

$= \left[100\right] - \left[10\right]$

$= 90$

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Find the area under the curve between the given x-values using the Fundamental Theorem of Integral Calculus.

#13) $f(x) = 6x^2 - 1 , x = 1, x = 2$

$A = \int_1^2 (6x^2-1)\,dx = (2x^3 - x)\Big|_1^2$

$= [2(2)^3 - (2)] - [2(1)^3 - (1)]$

$= [2(8) - 2] - [2(1) - 1]$

$= [16 - 2] - [2 - 1]$

$= [14] - [1]$

$A = 13\,un^2$

#15) $f(x) = \frac{1}{x} , x = 1, x = 100$

$A = \int_1^{100} \frac{1}{x}\,dx$

$= \ln|x|\Big|_1^{100}$

$= \ln|100| - \ln|1|$

$= \ln 100 - 0$

$A = \ln 100\ un^2$

#14) $f(x) = 12e^{2x} , x = 0, x = 3$

$A = \int_0^3 12e^{2x}\,dx = 6e^{2x}\Big|_0$

$= 6e^{2(3)} - 6e^{2(0)}$

$= 6e^6 - 6e^0$

$A = (6e^6 - 6)un^2$

#16) $f(x) = e^{\frac{x}{2}} , x = 0, x = 4$

$A = \int_0^4 e^{\frac{1}{2}x}\,dx$

$= 2e^{\frac{1}{2}x}\Big|_0^4$

$= 2e^{\frac{1}{2}(4)} - 2e^{\frac{1}{2}(0)}$

$= 2e^2 - 2e^0$

$A = (2e^2 - 2)\,un^2$

#17) A company's marginal cost function is $MC(x) = x^{-1/2} + 4$, where x is the number of units, and fixed costs are \$20,000. Find the cost function.

(0, 20,000)
(x, C(x))

$C(x) = \int (x^{-\frac{1}{2}} + 4)\,dx$

$C(x) = 2x^{\frac{1}{2}} + 4x + C$

$20{,}000 = 2\sqrt{0} + 4(0) + C$

$20{,}000 = C$

$C(x) = 2\sqrt{x} + 4x + C$

#18) A homeowner installs a solar heating system, which is expected to generate savings at the rate of $200e^{0.1t}$ dollars per year, where t is the number of years since the system was installed. (years, savings) (0, 0)

a. Find a formula for the total savings in the first t years.

$\text{Savings} = \int 200e^{0.1t}\,dt$

$S = 2000e^{0.1t} + C$

$0 = 2000e^{0.1(0)} + C$

$0 = 2000e^{0} + C$

$0 = 2000(1) + C$

$-2000 = C$

$S = 2000e^{0.1t} - 2000$

b. If the system originally cost \$1500, when will it "pay for itself"?

$1500 = 2000e^{0.1t} - 2000$

$3500 = 2000e^{0.1t}$

$\frac{3500}{2000} = e^{0.1t}$

$\ln\left(\frac{7}{4}\right) = \ln e^{0.1t}$

$\ln\left(\frac{7}{4}\right) = 0.1t$

$10\ln\left(\frac{7}{4}\right) = t$

$5.6 \text{ years} \approx t$

#19) A flu epidemic hits a college community, beginning with five cases on day $t = 0$. The rate of growth of the epidemic (new cases per day) is given by $r(t) = 18e^{0.05t}$, where t is the number of days since the epidemic began. (days, cases) (0, 5)

a. Find a formula for the total number of cases of flu in the first t days.

$\text{Total Cases} = \int 18e^{0.05t}\,dt$

$= 18(20)e^{0.05t} + C$

$TC = 360e^{0.05t} + C$

$5 = 360e^{0.05(0)} + C$

$5 = 360e^{0} + C$

$5 = 360(1) + C$

$-355 = C$

$TC = 360e^{0.05t} - 355$

b. Use your answer to part (a) to find the total number of cases in the first 20 days.

$TC(20) = 360e^{0.05(20)} - 355$

$= 360e^{1} - 355$

$TC(20) \approx 624 \text{ cases}$

#20) A student can memorize foreign vocabulary words at the rate of $\frac{2}{\sqrt[3]{t}}$ words per minutes, where t is the number of minutes since the studying began. Find the number of words that can be memorized in the first 8 minutes.

(minute, vocab words)

$\text{Words} = \int_0^8 2t^{-1/3}\,dt$

$= 2\left(\frac{3}{2}\right)t^{2/3}\Big|_0^8$

$= 3\sqrt[3]{t^2}\Big|_0^8$

$= 3(\sqrt[3]{8})^2 - 3(\sqrt[3]{0})^2$

$= 3(2)^2 - 3(0)^2$

$= 3(4) - 0$

$\text{Words} = 12$

#21) A company's marginal cost function is $MC(x) = x^{-1/2} + 4$ dollars, where *x* is the number of units. Find the total cost of the first 400 units (units *x = 0* to *x = 400*).

$$C(x) = \int_0^{400} \left(x^{-\frac{1}{2}} + 4\right) dx$$

$$= \left(2x^{\frac{1}{2}} + 4x\right)\Big|_0^{400}$$

$$= \left[2\sqrt{400} + 4(400)\right] - \left[2\sqrt{0} - 4(0)\right]$$

$$= \left[2(20) + 1600\right] - \left[0 - 0\right]$$

$$= 40 + 1600$$

$$C(x) = \$1640$$

#22) Find the area bounded by the curves.
$f(x) = x^2 + 3x$ and $f(x) = 3x + 1$

Cross? Yes @ -1, 1

$x^2 + 3x = 3x + 1$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x - 1 = 0$ | $x + 1 = 0$

$x = 1$ | $x = -1$

Upper / Lower

$0 \in (-1, 1)$

$y = x^2 + 3x$	$y = 3x + 1$
$y = (0)^2 + 3(0)$	$y = 3(0) + 1$
$y = 0$	$y = 1$
Lower	Upper

$$A = \int_{-1}^{1} \left[(3x+1) - (x^2 + 3x)\right] dx$$

$$= \int_{-1}^{1} \left[1 - x^2\right] dx$$

$$= \left[x - \tfrac{1}{3}x^3\right]\Big|_{-1}^{1}$$

$$= \left[1 - \tfrac{1}{3}(1)^3\right] - \left[(-1) - \tfrac{1}{3}(-1)^3\right]$$

$$= \left[1 - \tfrac{1}{3}(1)\right] - \left[-1 - \tfrac{1}{3}(-1)\right]$$

$$= \left[1 - \tfrac{1}{3}\right] - \left[-1 + \tfrac{1}{3}\right]$$

$$= 1 + 1 - \tfrac{1}{3} - \tfrac{1}{3}$$

$$= 2 - \tfrac{2}{3}$$

$$= \tfrac{6}{3} - \tfrac{2}{3}$$

$$A = \tfrac{4}{3}\ un^2$$

#23) Find the area bounded by the curves.

$y = x^2$ and $y = x$

cross? Yes @ -1, 0, 1

$x^3 + x^2 = x^2 + x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0$

$x = 0 \quad \{ x - 1 = 0, \; x = 1 \quad \} x + 1 = 0, \; x = -1$

Upper/Lower

$-\frac{1}{2} \in (-1, 0)$

$y = x^3 + x^2$	$y = x^2 + x$
$y = (-\frac{1}{2})^3 + (-\frac{1}{2})^2$	$y = (-\frac{1}{2})^2 + (-\frac{1}{2})$
$y = -\frac{1}{8} + \frac{1}{4}$	$y = \frac{1}{4} - \frac{2}{4}$
$y = -\frac{1}{8} + \frac{2}{8}$	$y = -\frac{1}{4}$
$y = \frac{1}{8}$	
upper	Lower

Upper/Lower

$\frac{1}{2} \in (0, 1)$

$y = x^3 + x^2$	$y = x^2 + x$
$y = (\frac{1}{2})^3 + (\frac{1}{2})^2$	$y = (\frac{1}{2})^2 + (\frac{1}{2})$
$y = \frac{1}{8} + \frac{1}{4}$	$y = \frac{1}{4} + \frac{2}{4}$
$y = \frac{1}{8} + \frac{2}{8}$	$y = \frac{3}{4}$
$y = \frac{3}{8}$	
Lower	upper

$$A = \int_{-1}^{0} [(x^3 + x^2) - (x^2 + x)]\,dx + \int_{0}^{1} [(x^2 + x) - (x^3 + x^2)]\,dx$$

$$= \int_{-1}^{0} [x^3 - x]\,dx + \int_{0}^{1} [x - x^3]\,dx$$

$$= \left[\frac{1}{4}x^4 - \frac{1}{2}x^2\right]\Big|_{-1}^{0} + \left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]\Big|_{0}^{1}$$

$$= \left[\frac{1}{4}(0)^4 - \frac{1}{2}(0)^2\right] - \left[\frac{1}{4}(-1)^4 - \frac{1}{2}(-1)^2\right] + \left[\frac{1}{2}(1)^2 - \frac{1}{4}(1)^4\right] - \left[\frac{1}{2}(0)^2 - \frac{1}{4}(0)^4\right]$$

$$= [\quad 0 \quad] - \left[\frac{1}{4}(1) - \frac{1}{2}(1)\right] + \left[\frac{1}{2}(1) - \frac{1}{4}(1)\right] - [\quad 0 \quad]$$

$$= -\left[\frac{1}{4} - \frac{2}{4}\right] + \left[\frac{2}{4} - \frac{1}{4}\right]$$

$$= -\left[-\frac{1}{4}\right] + \left[\frac{1}{4}\right]$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$A = \frac{1}{2} \text{ un}^2$$

Find the average value of the function on the given interval.

#24) $f(x) = \frac{1}{x}$ on [1, 4]

$$AV = \frac{1}{4-1} \int_{1}^{4} \frac{1}{x}\,dx$$

$$= \frac{1}{3} \ln|x| \Big|_{1}^{4}$$

$$= \left[\frac{1}{3} \ln|4|\right] - \left[\frac{1}{3} \ln|1|\right]$$

$$= \frac{1}{3} \ln 4 - \frac{1}{3}(0)$$

$$AV = \frac{1}{3} \ln 4$$