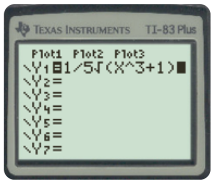
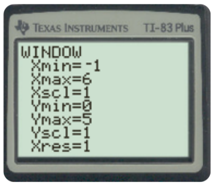
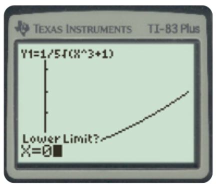
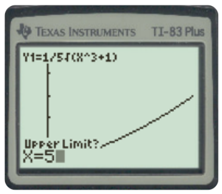
Find the average value of the function on the given interval.

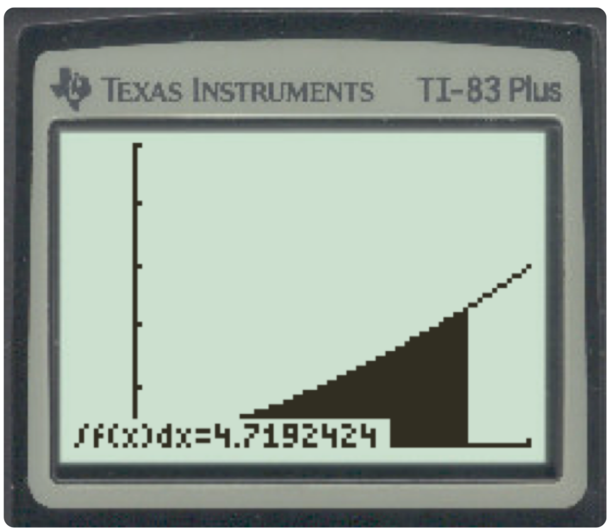
#25) $f(x) = \sqrt{x^3+1}$ *on* $[0, 5]$
(Use your calculator to find the area.)











#26) A deposit of \$3000 in a bank paying 6% interest compounded continuously will grow to $V(t) = 3000e^{0.06t}$ dollars after t years. Find the average value during the first 20 years (*t = 0* to *t = 20*).

$$AV = \frac{1}{20-0}\int_0^{20} 3000e^{0.06t}\,dt$$

$$= \frac{1}{\cancel{20}}\,\cancel{3000}\left(\frac{50}{\cancel{3}}\right)e^{0.06t}\Big|_0^{20}$$

$$= 2500e^{0.06t}\Big|_0^{20}$$

$$= 2500e^{0.06(20)} - 2500e^{0.06(0)}$$

$$= 2500e^{1.2} - 2500e^{0}$$

$$= 2500e^{1.2} - 2500$$

$$AV \approx \$5800.29$$

#27) Use the Riemann Sum by hand to approximate the area under the curve $f(x) = \sqrt{x}$ from 1 to 5 using 3 left rectangles with equal bases.

$a = 1$

$b = 5$

$\boxed{\Delta x = \frac{b-a}{n} = \frac{5-1}{3} = \frac{4}{3}}$

$x_1 = 1 = \frac{3}{3}$

$x_2 = 1 + \frac{4}{3} = \frac{3}{3} + \frac{4}{3} = \frac{7}{3}$

$x_3 = \frac{7}{3} + \frac{4}{3} = \frac{11}{3}$

$A \approx \int_1^5 \sqrt{x}\,dx$

$\approx \Delta x f(x_1) + \Delta x f(x_2) + \Delta x f(x_3)$

$\approx \frac{4}{3} f(1) + \frac{4}{3} f(\frac{7}{3}) + \frac{4}{3} f(\frac{11}{3})$

$\approx \frac{4}{3}\sqrt{1} + \frac{4}{3}\sqrt{\frac{7}{3}} + \frac{4}{3}\sqrt{\frac{11}{3}}$

$A \approx 5.92 \text{ un}^2$

Find each integral by substitution or state that it cannot be evaluated by our substitution formulas.

#28) $\int x^3 \sqrt[3]{x^4 - 1}dx$

$u = x^4 - 1$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\frac{du}{4x^3} dx$

$= \int \cancel{x^3} \sqrt[3]{u} \frac{du}{4\cancel{x^3}}$

$= \frac{1}{4} \int u^{\frac{1}{3}} du$

$= \frac{1}{4}\left(\frac{3}{4}\right) u^{\frac{4}{3}} + C$

$= \frac{3}{16} \sqrt[3]{(x^4-1)^4} + C$

#29) $\int \frac{x}{\sqrt{9+x^2}} dx$

$u = 9 + x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \int \frac{\cancel{x}}{u^{\frac{1}{2}}} \frac{du}{2\cancel{x}}$

$= \frac{1}{2} \int u^{-\frac{1}{2}} du$

$= \frac{1}{2}(2) u^{\frac{1}{2}} + C$

$= \sqrt{9+x^2} + C$

Find each definite integral. (A calculator may only be used to check your answer.)

#30) $\int_0^4 \frac{w}{\sqrt{25-w^2}}\,dw$

$u = 25 - w^2$
$\frac{du}{dw} = -2w$
$du = -2w\,dw$
$\frac{du}{-2w} = dw$

$$= \int_{w=0}^{w=4} \frac{\cancel{w}}{\sqrt{u}}\left(\frac{du}{-2\cancel{w}}\right)$$
$$= \frac{1}{-2}\int_{w=0}^{w=4} \frac{1}{\sqrt{u}}\,du$$
$$= -u^{\frac{1}{2}}\Big|_{w=0}^{w=4}$$
$$= -\sqrt{25-w^2}\Big|_0^4$$
$$= -\left[\sqrt{25-(4)^2} - \sqrt{25-(0)^2}\right]$$
$$= -\left[\sqrt{25-16} - \sqrt{25}\right]$$
$$= -\left[\sqrt{9} - 5\right]$$
$$= -[3-5]$$
$$= -[-2]$$
$$= 2$$

#31) Find the average value on the given interval.
$f(x) = xe^{-x^2}$ on $[0, 2]$

$u = -x^2$
$\frac{du}{dx} = -2x$
$du = -2x\,dx$
$\frac{du}{-2x} = dx$

$$AV = \frac{1}{2-0}\int_0^2 xe^{-x^2}\,dx$$
$$= \frac{1}{2}\int_{x=0}^{x=2} \cancel{x}\, e^u \frac{du}{-2\cancel{x}}$$
$$= -\frac{1}{4}\int_{x=0}^{x=2} e^u\,du$$
$$= -\frac{1}{4}e^u\Big|_{x=0}^{x=2}$$
$$= -\frac{1}{4}e^{-x^2}\Big|_0^2$$
$$= \left[-\frac{1}{4}e^{-(2)^2}\right] - \left[-\frac{1}{4}e^{-(0)^2}\right]$$
$$= -\frac{1}{4}e^{-4} + \frac{1}{4}e^0$$
$$AV = -\frac{1}{4}e^{-4} + \frac{1}{4}$$

#32) A company's marginal cost function is $MC(x) = \frac{1}{\sqrt{2x+9}}$ and fixed costs are \$100. Find the cost function.

$(0, \$100)$

$u = 2x+9$
$\frac{du}{dx} = 2$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$$C(x) = \int (2x+9)^{-\frac{1}{2}}\,dx$$
$$= \int u^{-\frac{1}{2}}\left(\frac{1}{2}du\right)$$
$$= \frac{1}{2}\int u^{-\frac{1}{2}}\,du$$
$$= \frac{1}{2}(2)u^{\frac{1}{2}} + C$$

$$C(x) = \sqrt{2x+9} + C$$
$$100 = \sqrt{2(0)+9} + C$$
$$100 = \sqrt{9} + C$$
$$100 = 3 + C$$
$$97 = C$$

$$C(x) = \sqrt{2x+9} + 97$$

#33) Find the area under the curve between the given x –values. $f(x) = \frac{x^2+6x}{\sqrt[3]{x^3+9x^2+17}}$ from $x = 1$ to $x = 3$.

$$A = \int_1^3 \frac{\cancel{x^2+6x}}{u^{1/3}} \frac{du}{3\cancel{(x^2+6x)}}$$

$$u = x^3+9x^2+17$$
$$\frac{du}{dx} = 3x^2+18x$$
$$du = (3x^2+18x)dx$$
$$\frac{du}{3(x^2+6x)} = dx$$

$$= \frac{1}{3}\int_{x=1}^{x=3} u^{-1/3}\,du$$

$$= \frac{1}{3}\left(\frac{3}{2}\right)u^{2/3}\Big|_{x=1}^{x=3}$$

$$= \frac{1}{2}\left(\sqrt[3]{x^3+9x^2+17}\right)^2\Big|_1^3$$

$$= \left[\frac{1}{2}\left(\sqrt[3]{(3)^3+9(3)^2+17}\right)^2\right] - \left[\frac{1}{2}\left(\sqrt[3]{(1)^3+9(1)^2+17}\right)^2\right.$$

$$= \frac{1}{2}\left(\sqrt[3]{27+9(9)+17}\right)^2 - \frac{1}{2}\left(\sqrt[3]{1+9(1)+17}\right)^2$$

$$= \frac{1}{2}\left(\sqrt[3]{27+81+17}\right)^2 - \frac{1}{2}\left(\sqrt[3]{1+9+17}\right)^2$$

$$= \frac{1}{2}\left(\sqrt[3]{125}\right)^2 - \frac{1}{2}\left(\sqrt[3]{27}\right)^2$$

$$= \frac{1}{2}(5)^2 - \frac{1}{2}(3)^2$$

$$= \frac{25}{2} - \frac{9}{2}$$

$$= \frac{16}{2}$$

$$A = 8\ un^2$$

#34) $\int 9\cos(x)\tan(x)\,dx$

$$= \int 9\cancel{\cos(x)}\frac{\sin(x)}{\cancel{\cos(x)}}\,dx$$

$$= \int 9\sin(x)\,dx$$

$$= -9\cos(x) + C$$

#35) $\int \sqrt{\sec^2(x) - 1}\,dx$

$$= \int \sqrt{\tan^2(x)}\,dx$$

$$= \int \tan(x)\,dx$$

$$= \ln|\sec(x)| + C$$

#36) $\int \left[\frac{\sec(x)}{\cos(x)} - \frac{\tan(x)}{\cot(x)}\right] dx$

$= \int \left[\sec^2(x) - \tan^2(x)\right] dx$

$= \int 1 \, dx$

$= x + C$

#37) $\int \left[\frac{1 - 2\cos^2(x)}{\sin(x)\cos(x)}\right] dx$

$= \int \left[\frac{\sin^2(x) + \cos^2 x - 2\cos^2(x)}{\sin(x)\cos(x)}\right] dx$

$= \int \frac{\sin^2(x) - \cos^2(x)}{\sin(x)\cos(x)} dx$

$= \int \left[\frac{\sin^2(x)}{\sin(x)\cos(x)} - \frac{\cos^2(x)}{\sin(x)\cos(x)}\right] dx$

$= \int \left[\frac{\sin(x)}{\cos(x)} - \frac{\cos(x)}{\sin(x)}\right] dx$

$= \int \left[\tan(x) - \cot(x)\right] dx$

$= \ln|\sec(x)| - \ln|\sin(x)| + C$

#38) $\int \sin\left(\frac{\pi}{3}x\right) dx$

$u = \frac{\pi}{3}x$

$\frac{du}{dx} = \frac{\pi}{3}$

$du = \frac{\pi}{3} dx$

$\frac{3}{\pi} du = dx$

$= \int \sin(u)\left(\frac{3}{\pi} du\right)$

$= \frac{3}{\pi} \int \sin(u) du$

$= -\frac{3}{\pi} \cos(u) + C$

$= -\frac{3}{\pi} \cos\left(\frac{\pi}{3}x\right) + C$

#39) $\int \sec(x) \ln|\sec(x) + \tan(x)| \, dx$

$u = \ln|\sec(x) + \tan(x)|$

$\frac{du}{dx} = \sec(x)$

$du = \sec(x) dx$

$\frac{du}{\sec(x)} = dx$

$= \int \cancel{\sec(x)} \cdot u \left(\frac{du}{\cancel{\sec(x)}}\right)$

$= \int u \, du$

$= \frac{1}{2} u^2 + C$

$= \frac{1}{2}\left[\ln|\sec(x) + \tan(x)|\right]^2 + C$

#40) $\int \csc(x)\cot(x)\csc^2(x)\,dx$

$u = \csc(x)$

$\frac{du}{dx} = -\csc(x)\cot(x)$

$du = -\csc(x)\cot(x)dx$

$\frac{du}{-\csc(x)\cot(x)} = dx$

$= \int \cancel{\csc(x)\cot(x)}\, u^2 \left(\frac{du}{-\cancel{\csc(x)\cot(x)}}\right)$

$= -\int u^2 du$

$= -\frac{1}{3}u^3 + C$

$= -\frac{1}{3}\csc^3(x) + C$

#41) $\int \frac{\sec(x)}{\cos(x)}dx$

$= \int \sec^2(x)\,dx$

$= \tan(x) + C$

#1: $4x^{\frac{3}{2}} - 5x + C$
#2: $2x^{\frac{5}{2}} - 3x^2 + C$
#3: $\frac{1}{3}x^3 - 16x + C$
#4: $x^3 + x^2 + 4x + C$
#5: $2e^{\frac{x}{2}} + C$
#6: $4\ln|x| + C$
#7: $3x^3 + 2\ln|x| + 2e^{3x}$
#8: $-x^{-1} + \ln|x| - e^{-x} + C$
#9: 36
#10: 4
#11: 2843.09
#12: 90
#13: $13\ un^2$
#14: $(6e^6 - 6)un^2$
#15: $\ln 100\ un^2$
#16: $(2e^2 - 2)un^2$
#17: $C(x) = 2\sqrt{x} + 4x + 20{,}000$
#18: a.) $f(t) = 2000e^{0.1t} - 2000$
b.) It will take about 5.6 years for it to pay for itself.

#19: a.) $f(t) = 360e^{0.05t} - 355$
b.) There is a total of about 624 cases of the epidemic in the first 20 days.

#20: About 12 words can be memorized in the first 8 minutes.

#21: The total cost of the first 400 units is $1640.00
#22: $\frac{4}{3}\ un^2$
#23: $\frac{1}{6}\ un^2$
#24: $\frac{1}{3}\ln 4$
#25: 4.72
#26: The average value of the account during the first 20 years is $5800.29.
#27: About 5.92 $un.^2$
#28: $\frac{3}{16}(x^4 - 1)^{\frac{4}{3}} + C$
#29: $(9 + x^2)^{\frac{1}{2}} + C$

#30: 2
#31: $-\frac{1}{4}e^{-4} + \frac{1}{4}$
#32: $C(x) = (2x + 9)^{\frac{1}{2}} + 97$
#33: $8\ un.^2$

#34) - #41) Tell me to get my act together