Basic Derivative Rules
Chapter 2 Review

#1) Find the equation for the tangent line to the curve $f(x) = x^2 - 7x + 18$ at $x = 4$. Write the answer in slope-intercept form.

Point @ $x=4$

$f(x) = x^2 - 7x + 18$
$f(4) = (4)^2 - 7(4) + 18$
$= 16 - 28 + 18$
$= 16 - 10$
$f(4) = 6$
$(4, 6)$

Slope @ $x=4$

$f'(x) = 2x - 7$
$f'(4) = 2(4) - 7$
$= 8 - 7$
$f'(4) = 1$
$m = 1$

Point-slope form

$y - y_1 = m(x - x_1)$
$y - (6) = 1(x - (4))$
$y - 6 = x - 4$
$y = x + 2$

#2) In a psychology experiment, a person could memorize x words in $f(x) = 2x^2 - x$ seconds (for $0 \le x \le 10$).

a. Find $f'(x)$
b. Evaluate $f'(5)$
c. Interpret $f'(5)$ as an instantaneous rate of change in the proper units.

$f(x)$ = Seconds	x = words	f' = Seconds/word

a. $f'(x) = 4x - 1$

b. $f'(5) = 4(5) - 1$
$= 20 - 1$
$f'(5) = 19$

c. After memorizing 5 words, a person will take 19 seconds to memorize the next word.

#3) If $g(w) = \sqrt[3]{w} - \frac{1}{w}$ find $\frac{dg}{dw}$

$g(w) = w^{\frac{1}{3}} - w^{-1}$

$\frac{dg}{dw} = \frac{1}{3}w^{-2/3} + w^{-2}$

$\frac{dg}{dw} = \frac{1}{3\sqrt[3]{w^2}} + \frac{1}{w^2}$

#4) If $f(x) = x^4$ find $\left.\frac{df}{dx}\right|_{x=-2}$

$\left.\frac{d}{dx}(x^4)\right|_{x=-2} = \left.4x^3\right|_{x=-2}$
$= 4(-2)^3$
$= 4(-8)$
$\left.\frac{d}{dx}(x^4)\right|_{x=-2} = -32$

#5) Why is the derivative referred to as an "instantaneous" rate of change rather than just an "average" rate of change?

An average rate of change is just the slope formula. It is how you calculate the slope of a secant line which requires two points, or two moments in time. Because you are measuring at two points, you are finding the average change that happens between the points.

The derivative is the slope formula with "the limit as h approaches 0" in front of it. By adding the limit as h approaches 0 to the slope formula, the distance between the two points needed to find the slope shrinks down to 0, giving the instantaneous rate of change at one moment in time.

#6) The number of ants noshing on some peaches at a picnic is $A(x) = 8000\sqrt{x} - 6000\sqrt[3]{x}$ ants, where x is the minutes since the first ant crashed the picnic

a. Find $A'(x)$.
b. Find $A'(64)$.
c. Interpret your answer from (b)

A = # ants	x = minutes	A' = ants/min

a. $A'(x) = 8000\left(\frac{1}{2}\right)x^{-\frac{1}{2}} - 6000\left(\frac{1}{3}\right)x^{-2/3}$

$= \frac{4000}{\sqrt{x}} - \frac{2000}{\sqrt[3]{x^2}}$

$A'(x) = \frac{4000}{\sqrt{x}} - \frac{2000}{\sqrt[3]{x^2}}$

b. $A'(64) = \frac{4000}{\sqrt{64}} - \frac{2000}{(\sqrt[3]{64})^2}$

$= \frac{4000}{8} - \frac{2000}{(4)^2}$

$= 500 - \frac{2000}{16}$

$= 500 - 125$

$A'(64) = 375$ Ants/min

c. Sixty four minutes after the first ant showed up, the number of ants that are at the picnic are increasing by 375 ants per minute.

#7) Differentiate.

$$f(x) = (x^2 + 2x)(2x + 1)$$

$f'(x) = (x^2+2x)'(2x+1) + (x^2+2x)(2x+1)'$

$= (2x+2)(2x+1) + (x^2+2x)(2)$

$= 4x^2 + 6x + 2 + 2x^2 + 4x$

$f'(x) = 6x^2 + 10x + 2$

#8) Differentiate $f(x) = \frac{x^4+x^2+1}{x^2+1}$

$f'(x) = \frac{(x^4+x^2+1)'(x^2+1) - (x^4+x^2+1)(x^2+1)'}{(x^2+1)^2}$

$= \frac{(4x^3+2x)(x^2+1) - (x^4+x^2+1)(2x)}{(x^2+1)^2}$

$= \frac{4x^5 + 6x^3 + \cancel{2x} - 2x^5 - 2x^3 - \cancel{2x}}{(x^2+1)^2}$

$= \frac{2x^5 + 4x^3}{(x^2+1)^2}$

$f'(x) = \frac{2x^3(x^2+2)}{(x^2+1)^2}$

#9) Differentiate $f(x) = \frac{2e^{7x}}{\ln(x)}$

$f'(x) = \frac{(2e^{7x})' \cdot \ln(x) - 2e^{7x} \cdot [\ln(x)]'}{[\ln(x)]^2}$

$= \frac{14e^{7x}\ln(x) - 2e^{7x} \cdot \frac{1}{x}}{\ln^2(x)}$

$f'(x) = \frac{14e^{7x}\ln(x) - \frac{2e^{7x}}{x}}{\ln^2(x)}$

#10) Differentiate $f(x) = x^3e^x$

$f'(x) = (x^3)'e^x + x^3(e^x)'$

$f'(x) = 3x^2e^x + x^3e^x$