Basic Derivative Rules
Chapter 2 Review II
Optional

#1) Find the equation for the tangent line to the curve $f(x) = 3x^2 - 2x + 4$ at $x = 1$. Write the answer in slope-intercept form.

Point @ $x=1$

$f(x) = 3x^2 - 2x + 4$
$f(1) = 3(1)^2 - 2(1) + 4$
$= 3(1) - 2 + 4$
$= 3 + 2$
$f(1) = 5$
$(1, 5)$

Slope @ $x=1$

$f'(x) = 6x - 2$
$f'(1) = 6(1) - 2$
$= 6 - 2$
$f'(1) = 4$
$m = 4$

Point-Slope Form

$y - y_1 = m(x - x_1)$
$y - (5) = 4(x - (1))$
$y - 5 = 4x - 4$
$y = 4x + 1$

#2) The temperature of a patient in a hospital on day x of an illness is given by in $T(x) = -x^2 + 5x + 100$ degrees Fahrenheit (for $1 < x < 5$).

a. Find $T'(x)$
b. Use your answer from part (a) to find the instantaneous rate of change of temperature on day 3
c. Interpret your answer from part (b)

$T(x)$ = degrees F	x = day	$T'(x) = {}^{\circ}F/\text{day}$

a. $T' = -2x + 5$

b. $T'(3) = -2(3) + 5$
$= -6 + 5$
$T'(3) = -1\ {}^{\circ}F/\text{day}$

c. On day 3 in a hospital, the temperature of the patient is decreasing by 1° per day.

#3) If $f(p) = \frac{10}{p} - 9\sqrt[3]{p^5} + 17$ find $\frac{df}{dp}$

$f(p) = 10p^{-1} - 9p^{5/3} + 17$

$\frac{df}{dp} = -10p^{-2} - 9\left(\frac{5}{3}\right)p^{2/3}$

$\frac{df}{dp} = \frac{-10}{p^2} - 15\sqrt[3]{p^2}$

#4) If $f(x) = \frac{54}{\sqrt{x}} + 12\sqrt{x}$ find $\left.\frac{df}{dx}\right|_{x=9}$

$\left.\frac{d}{dx}\left(54x^{-\frac{1}{2}} + 12x^{\frac{1}{2}}\right)\right|_{x=9} = \left.\left(-27x^{-3/2} + 6x^{-\frac{1}{2}}\right)\right|_{x=9}$

$= \left.\left(\frac{-27}{(\sqrt{x})^3} + \frac{6}{\sqrt{x}}\right)\right|_{x=9}$

$= \frac{-27}{(\sqrt{9})^3} + \frac{6}{\sqrt{9}}$

$= \frac{-27}{(3)^3} + \frac{6}{3}$

$= \frac{-27}{27} + 2$

$= -1 + 2$

$\left.\frac{d}{dx}\left(54x^{-\frac{1}{2}} + 12x^{\frac{1}{2}}\right)\right|_{x=9} = 1$

#5) Why is the derivative referred to as an "instantaneous" rate of change rather than just an "average" rate of change?

An average rate of change is just the slope formula. It is how you calculate the slope of a secant line which requires two points, or two moments in time. Because you are measuring at two points, you are finding the average change that happens between the points.

The derivative is the slope formula with "the limit as h approaches 0" in front of it. By adding the limit as h approaches 0 to the slope formula, the distance between the two points needed to find the slope shrinks down to 0, giving the instantaneous rate of change at one moment in time.

#6) George finds the total paints chips he can eat after finishing a gallon of milk is $P(x) = 0.02x^{3/2} + 3000$ chips, where x is the seconds after drinking the milk.

a. Find $P'(x)$.
b. Find $P'(10{,}000)$.
c. Interpret your answer from (b)

P = paint chips | x = seconds | P' = chips/second

a. $P'(x) = 0.02\left(\frac{3}{2}\right)x^{\frac{1}{2}}$

$P'(x) = 0.03\sqrt{x}$

b. $P'(10{,}000) = 0.03\sqrt{10{,}000}$

$= 0.03(100)$

$P'(10{,}000) = 3$ chips/second

c. Ten thousand seconds after drinking a gallon of milk, George is eating 3 paint chips per second.

#7) Differentiate $f(t) = 6t^{4/3}(3t^{2/3} + 1)$

$f(t) = 18t^{6/3} + 6t^{4/3}$

$f(t) = 18t^2 + 6t^{4/3}$

$f'(t) = 36t + 6\left(\frac{4}{3}\right)t^{1/3}$

$f'(t) = 36t + 8t^{1/3}$

#8) Differentiate $f(x) = \frac{x^5+x^3+x}{x^3+x} = \frac{\cancel{x}(x^4+x^2+1)}{\cancel{x}(x^2+1)}$

$f(x) = \frac{x^4+x^2+1}{x^2+1}$

$f'(x) = \frac{(x^4+x^2+1)'(x^2+1) - (x^4+x^2+1)(x^2+1)'}{(x^2+1)^2}$

$= \frac{(4x^3+2x)(x^2+1) - (x^4+x^2+1)(2x)}{(x^2+1)^2}$

$= \frac{4x^5+6x^3+\cancel{2x}-2x^5-2x^3-\cancel{2x}}{(x^2+1)^2}$

$= \frac{2x^5+4x^3}{(x^2+1)^2}$

$f'(x) = \frac{2x^3(x^2+2)}{(x^2+1)^2}$

#9) Differentiate $f(x) = x\ln x - x$

$f'(x) = x'\ln(x) + x\cdot[\ln(x)]' - 1$

$= 1\cdot\ln(x) + x\cdot\frac{1}{x} - 1$

$= \ln(x) + 1 - 1$

$f'(x) = \ln(x)$

#10) Differentiate $f(x) = e^x + x^e$

$f'(x) = e^x + ex^{e-1}$