Limits & Continuity
Chapter 1 Review II
OPTIONAL

Find the following limits *without* using a graphing calculator or making tables.

#1) $\lim_{x\to 7}\frac{x^2-x}{2x-7}$ $=\frac{(7)^2-(7)}{2(7)-7}$

$=\frac{49-7}{14-7}$

$=\frac{42}{7}$

$\lim_{x\to 7}\frac{x^2-x}{2x-7}=6$

#2) $\lim_{s\to 4}(s^{\frac{3}{2}}-3s^{\frac{1}{2}})$ $=(\sqrt{(4)})^3-3\sqrt{(4)}$

$=(2)^3-3(2)$

$=8-6$

$\lim_{s\to 4}(s^{3/2}-3s^{1/2})=2$

#3) $\lim_{x\to 0}\frac{x^2-x}{x^2+x}$ $=\lim_{x\to 0}\frac{x(x-1)}{x(x+1)}$

$=\lim_{x\to 0}\frac{x-1}{x+1}$

$=\frac{(0)-1}{(0)+1}$

$=\frac{-1}{1}$

$\lim_{x\to 0}\frac{x^2-x}{x^2+x}=-1$

#4) $\lim_{h\to 0}\frac{5x^4h-9xh^2}{h}$ $=\lim_{h\to 0}\frac{h(5x^4-9xh)}{h}$

$=\lim_{h\to 0}(5x^4-9xh)$

$=5x^4-9x(0)$

$\lim_{h\to 0}\frac{5x^4h-9xh^2}{h}=5x^4$

Answer each question concerning piecewise functions.

#5) $f(x)=\begin{cases}5-x, & if\ x<4\\ 2x-5, & if\ x\geq 4\end{cases}$

a. $\lim_{x\to 4^-}f(x)=$ $\lim_{x\to 4^-}(5-x)$

$=5-(4)$

$\lim_{x\to 4^-}f(x)=1$

b. $\lim_{x\to 4^+}f(x)=$ $\lim_{x\to 4^+}(2x-5)$

$=2(4)-5$

$=8-5$

$\lim_{x\to 4^+}f(x)=3$

c. $\lim_{x\to 4}f(x)=$ dne.

#6) $f(x)=\begin{cases}2-x, & if\ x<4\\ 2x-10, & if\ x\geq 4\end{cases}$

a. $\lim_{x\to 4^-}f(x)=$ $\lim_{x\to 4^-}(2-x)$

$=2-(4)$

$\lim_{x\to 4^-}f(x)=-2$

b. $\lim_{x\to 4^+}f(x)=$ $\lim_{x\to 4^+}(2x-10)$

$=2(4)-10$

$=8-10$

$\lim_{x\to 4^+}f(x)=-2$

c. $\lim_{x\to 4}f(x)=$ -2

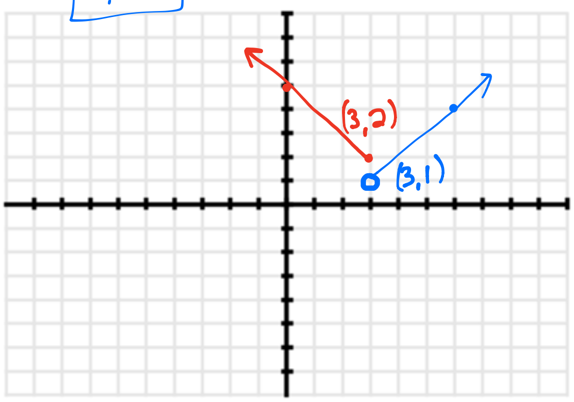
#7) For the following piecewise function:

$$f(x) = \begin{cases} 5 - x, & if\ x \leq 3 \\ x - 2, & if\ x > 3 \end{cases}$$

a. Draw its graph

x	5−x
3	2
0	5

x	x−2
3	1
6	4



b. Find the limits as x approaches 3 from the left.

$\lim_{x\to 3^-} f(x) = 2$

c. Find the limits as x approaches 3 from the right.

$\lim_{x\to 3^+} f(x) = 1$

d. Is it continuous at x = 3? If not, why?

No because $\lim_{x\to 3} f(x) = dne$

OR

No because of jump discontinuity

Find $f'(x)$ by using the definition of the derivative.

#8) $f(x) = 2x^2 - 5x + 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[2(x+h)^2 - 5(x+h) + 1] - [2x^2 - 5x + 1]}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2 + 2hx + h^2) - 5x - 5h + 1 - 2x^2 + 10x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 \cancel{-5x} - 5h \cancel{+1} \cancel{-2x^2} \cancel{+10x} \cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{4hx + 2h^2 - 5h}{h}$$

$$= \lim_{h\to 0} \frac{h(4x + 2h - 5)}{h}$$

$$= \lim_{h\to 0} (4x + 2h - 5)$$

$$= 4x + 2(0) - 5$$

$$f'(x) = 4x - 5$$

#9) $f(x) = -3x + 5$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[-3(x+h) + 5] - [-3x + 5]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-3x} - 3h \cancel{+5} \cancel{+3x} \cancel{-5}}{h}$$

$$= \lim_{h\to 0} \frac{-3h}{h}$$

$$= \lim_{h\to 0} -3$$

$$f'(x) = -3$$

Find $\frac{d}{dx} f(x)$ by using the definition of the derivative.

#10) $f(x) = \frac{1}{2x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x}{2(x+h)x} - \frac{(x+h)}{2x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x-x-h}{2x(x+h)}}{h} \quad \frac{\frac{1}{h}}{\frac{1}{h}}$$

$$= \lim_{h \to 0} \frac{-h}{2x(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{2x(x+h)}$$

$$= \frac{-1}{2x(x+(0))}$$

$$= \frac{-1}{2x(x)}$$

$$f'(x) = \frac{-1}{2x^2}$$

#11) Find the equation for the tangent line to the curve $f(x) = 2x^2 - 5x + 1$ at $x = 2$. Write your equation in slope-intercept form. (Use a graphing calculator to graph the curve with the tangent line to verify your answer.)

Slope of All tangent lines

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 - 5(x+h) + 1] - [2x^2 - 5x + 1]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - 5x - 5h + 1 - 2x^2 + 5x - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{5x} - 5h + \cancel{1} - \cancel{2x^2} + \cancel{5x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{4hx + 2h^2 - 5h}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$$

$$= \lim_{h \to 0} (4x + 2h - 5)$$

$$= 4x + 2(0) - 5$$

$$f'(x) = 4x - 5$$

Point @ $x=2$

$f(x) = 2x^2 - 5x + 1$
$f(2) = 2(2)^2 - 5(2) + 1$
$= 2(4) - 10 + 1$
$= 8 - 9$
$f(2) = -1$
$(2, -1)$

Slope @ $x=2$

$f'(x) = 4x - 5$
$f'(2) = 4(2) - 5$
$= 8 - 5$
$f'(2) = 3$
$m = 3$

Point-slope form

$y - y_1 = m(x - x_1)$
$y - (-1) = 3(x - (2))$
$y + 1 = 3x - 6$
$y = 3x - 7$

#12) Find the equation for the tangent line to the curve $f(x) = 3x^2 - 8x + 7$ at $x = 3$. Write your equation in slope-intercept form. (Use a graphing calculator to graph the curve with the tangent line to verify your answer.)

Slope of All tangent lines

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^2 - 8(x+h) + 7] - [3x^2 - 8x + 7]}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 8x - 8h + 7 - 3x^2 + 8x - 7}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{8x} - 8h + \cancel{7} - \cancel{3x^2} + \cancel{8x} - \cancel{7}}{h}$$

$$= \lim_{h \to 0} \frac{6hx + 3h^2 - 8h}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h - 8)}{h}$$

$$= \lim_{h \to 0} (6x + 3h - 8)$$

$$= 6x + 3(0) - 8$$

$$f'(x) = 6x - 8$$

Point @ $x=3$

$f(x) = 3x^2 - 8x + 7$
$f(3) = 3(3)^2 - 8(3) + 7$
$= 3(9) - 24 + 7$
$= 27 - 17$
$f(3) = 10$
$(3, 10)$

Slope @ $x=3$

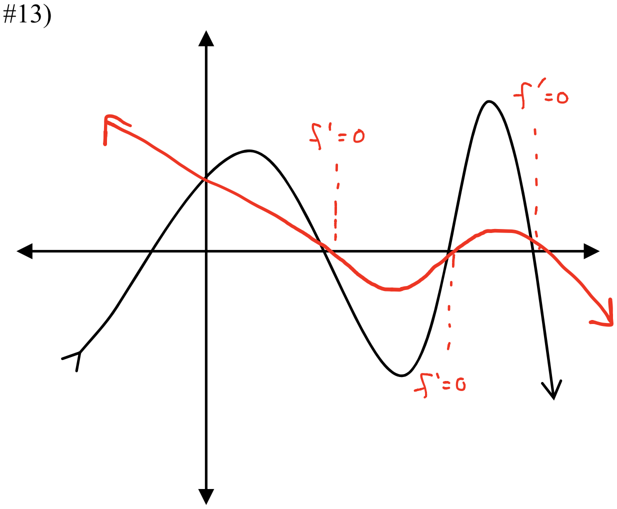
$f'(x) = 6x - 8$
$f'(3) = 6(3) - 8$
$= 18 - 8$
$f'(3) = 10$
$m = 10$

Point-slope form

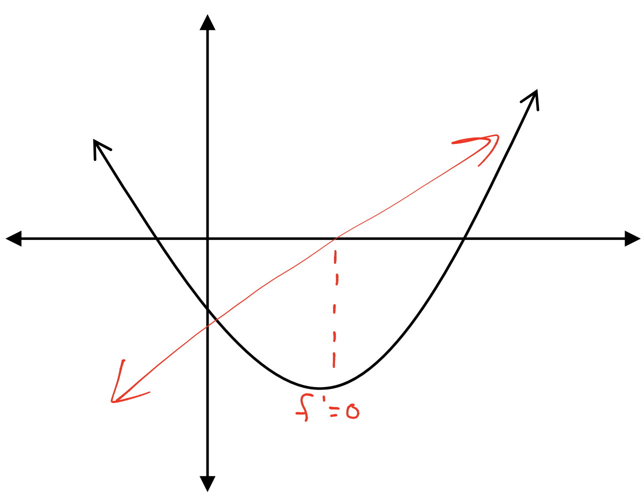
$y - y_1 = m(x - x_1)$
$y - (10) = 10(x - (3))$
$y - 10 = 10x - 30$
$y = 10x - 20$

Given the graph of a function, sketch in the graph of its derivative function.

#13)



#14)

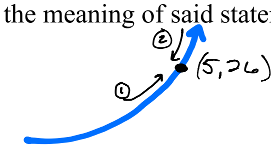


#15) Often times, problems will ask for the derivative without using the word "derivative". We have learned two interpretations of a derivative. What are these two interpretations?

Derivative means the slope of a tangent line.

Derivative means the instantaneous rate of change.

#16) $\lim_{x \to 5}(x^2+1)=26$ is read as x approaches 5 of x^2+1 is equal to 26. Use sentences and graphs to illustrate the meaning of said statement.

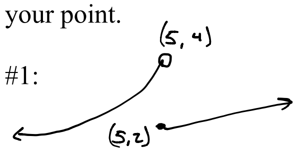


①As x gets closer and closer to 5 from the left, y gets closer and closer to 26.

② As x approaches 5 from the right y approaches 26.

#17) Give 2 specific scenarios of when a limit would not exist and explain why. You *may* use graphs to illustrate your point.

Scenario #1:



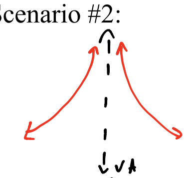
The limit would not exist at $x=5$.

As $x \to 5^-$ the limit is 4.

As $x \to 5^+$ the limit is 2.

The two-sided limit does not exist because the left and right limits do not agree

Scenario #2:



The limit does not exist @ $x=10$.
As $x \to 10$ from the left or the right, the y-value approaches infinity.

For a limit to exist, it must approach a single number.
(∞ is not a number)