Limits & Continuity
1.2 – Limits by Substitution

Some limits can be found by direct substitution, while others cannot. The "Rules for Limits" exist to help in determining which limits can be found by substitution.

Rules for Limits

For any constants a and c, and any positive integer n:

1. $\lim_{x \to c} a = a$

2. $\lim_{x \to c} x^n = c^n$

3. $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$ ($c > 0$ if n is even)

4. If $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ both exist, then

   a. $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

   b. $\lim_{x \to c}[f(x) - g(x)] = \lim_{x \to c} f(x) - \lim_{x \to c} g(x)$

   c. $\lim_{x \to c}[f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$

   d. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} \quad if \ \lim_{x \to c} g(x) \neq 0$

Summary of Rules of Limits

For functions composed of additions, subtractions, multiplications, divisions, powers, and roots, limits may be evaluated by direct substitution, provided that the resulting expression is defined.

$$\lim_{x \to c} f(x) = f(c)$$

Ex A: Finding Limits by Direct Substitution

#1) Find $\lim_{x \to 4} \sqrt{x}$.

Simply substitute c into x and simplify. As long as your answer is defined, this is the simplest way to evaluate a limit.

$\lim_{x \to 4} \sqrt{x} = \sqrt{4}$

$\lim_{x \to 4} \sqrt{x} = 2$

#2) Find $\lim_{x \to 6} \frac{x^2}{x+3}$.

$\lim_{x \to 6} \frac{x^2}{x+3} = \frac{(6)^2}{(6)+3}$

$= \frac{36}{9}$

$\lim_{x \to 6} \frac{x^2}{x+3} = 4$

#3) Find $\lim_{x \to 3}(2x^2 - 4x + 1)$.

$\lim_{x \to 3}(2x^2 - 4x + 1) = 2(3)^2 - 4(3) + 1$

$= 2(9) - 12 + 1$

$= 18 - 11$

$\lim_{x \to 3}(2x^2 - 4x + 1) = 7$

Limits & Continuity
1.2 – Limits by Substitution

Sometimes direct substitution into a quotient gives an undefined expression. If this happens, factoring, simplifying, and then using direct substitution may help.

Ex B: Finding Limits by Simplifying

#1) Find $\lim_{x\to 1} \frac{x^2-1}{x-1}$ and graph the function.

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1} (x+1)$$

$$= (1)+1$$

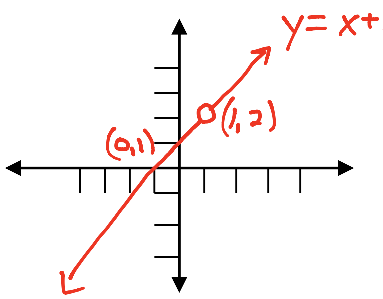
$$= 2$$

HolE $(1,2)$

$x-1=0$
$x=1$

$y=x+1$
$y=(1)+1$
$y=2$

$y=x+1$, hole $(1,2)$



#2) Find $\lim_{x\to 5} \frac{2x^2-10x}{x-5}$ and graph the function.

$$\lim_{x\to 5} \frac{2x^2-10x}{x-5} = \lim_{x\to 5} \frac{2x\cancel{(x-5)}}{\cancel{x-5}}$$

$$= \lim_{x\to 5} 2x$$

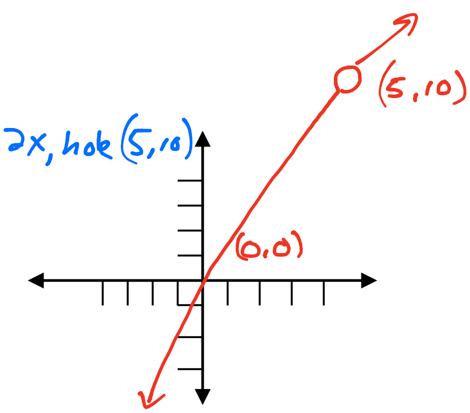
$$= 2(5)$$

$$= 10$$

HoLE $(5,10)$

$x-5=0$ $x=5$	$y=2x$ $y=2(5)$ $y=10$

$y=2x$, hole $(5,10)$



Finding each limit by substitution. You may have to simplify first.

#1) $\lim_{x\to 9}\sqrt{x+7} = \sqrt{(9)+7}$
$= \sqrt{16}$
$= 4$

#2) $\lim_{x\to 2}(4x^2-7x+1) = 4(2)^2 - 7(2) + 1$
$= 4(4) - 14 + 1$
$= 16 - 13$
$= 3$

#3) $\lim_{x\to 15}\frac{2x^2-30x}{x-15} = \lim_{x\to 15}\frac{2x\cancel{(x-15)}}{\cancel{x-15}}$
$= \lim_{x\to 15} 2x$
$= 2(15)$
$= 30$

#4) $\lim_{x\to -5}\sqrt{77} = \sqrt{77}$

#5) $\lim_{x\to 81}\left[(x-80)x^{1/2}\right] = [(81)-80]\,(81)^{\frac{1}{2}}$
$= [1]\cdot\sqrt{81}$
$= 1\cdot 9$
$= 9$

#6) $\lim_{h\to 0}(3x^2h - 3xh + 33) = 3x^2(0) - 3x(0) + 33$
$= 33$

#7) $\lim_{h\to 5}\left[\frac{x^2-5}{x-5}+h\right] = \frac{x^2-5}{x-5} + (5)$
$= \frac{x^2-5}{x-5} + 5$

#8) $\lim_{x\to -5}\frac{x+5}{x^2+7x+10} = \lim_{x\to -5}\frac{\cancel{x+5}}{\cancel{(x+5)}(x+2)}$
$= \lim_{x\to -5}\frac{1}{x+2}$
$= \frac{1}{(-5)+2}$
$= \frac{1}{-3}$

#9) $\lim_{x\to 0}\frac{6x^2-5x}{11x} = \lim_{x\to 0}\frac{\cancel{x}(6x-5)}{11\cancel{x}}$
$= \lim_{x\to 0}\frac{6x-5}{11}$
$= \frac{6(0)-5}{11}$
$= \frac{-5}{11}$

#10) $\lim_{h\to 0}\frac{3x^2h-12xh^2+4h^3}{h}$
$= \lim_{h\to 0}\frac{\cancel{h}(3x^2-12xh+4h^2)}{\cancel{h}}$
$= \lim_{h\to 0}(3x^2-12xh+4h^2)$
$= 3x^2 - 12x(0) + 4(0)^2$
$= 3x^2$

Limits & Continuity
1.2A – Limits by Substitution

Finding each limit by substitution. You may have to simplify first.

#11) $\lim_{x\to 9}\sqrt{x}$ $=\sqrt{(9)}$

$=3$

#12) $\lim_{x\to 2}(9x^2-8x+4)$ $=9(2)^2-8(2)+4$

$=9(4)-16+4$

$=36-12$

$=24$

#13) $\lim_{x\to 4}\frac{2x^2-15}{5x+1}$ $=\frac{2(4)^2-15}{5(4)+1}$

$=\frac{2(16)-15}{20+1}$

$=\frac{32-15}{21}$

$=\frac{17}{21}$

#14) $\lim_{x\to 2}\sqrt{11}$ $=\sqrt{11}$

#15) $\lim_{x\to 16}\left[(x+4)x^{-1/2}\right]$

$=\left[(16)+4\right](16)^{-\frac{1}{2}}$

$=\left[20\right]\cdot\frac{1}{\sqrt{16}}$

$=\frac{20}{4}$

$=5$

#16) $\lim_{h\to 0}(9x^2h-8xh+4)$ $=9x^2(0)-8x(0)+4$

$=4$

#17) $\lim_{x\to 5}\frac{x^2-25}{x-5}$ $=\lim_{x\to 5}\frac{\cancel{(x-5)}(x+5)}{\cancel{x-5}}$

$=\lim_{x\to 5}(x+5)$

$=(5)+5$

$=10$

#18) $\lim_{x\to -2}\frac{x+2}{x^2+7x+10}$ $=\lim_{x\to -2}\frac{\cancel{x+2}}{\cancel{(x+2)}(x+5)}$

$=\lim_{x\to -2}\frac{1}{x+5}$

$=\frac{1}{(-2)+5}$

$=\frac{1}{3}$

#19) $\lim_{x\to 0}\frac{x^3+x^2-x}{x^2+x}$ $=\lim_{x\to 0}\frac{\cancel{x}(x^2+x-1)}{\cancel{x}(x+1)}$

$=\lim_{x\to 0}\frac{x^2+x-1}{x+1}$

$=\frac{(0)^2+(0)-1}{(0)+1}$

$=\frac{-1}{1}$

$=-1$

#20) $\lim_{h\to 0}\frac{8x^2h+3xh^2+h^3}{h}$

$=\lim_{h\to 0}\frac{h(8x^2+3xh+h^2)}{h}$

$=\lim_{h\to 0}(8x^2+3xh+h^2)$

$=8x^2+3x(0)+(0)^2$

$=8x^2$