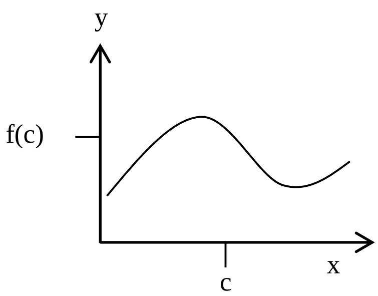
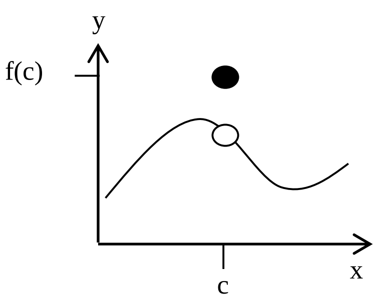
Limits & Continuity
1.4 – Continuity

Continuity from PreCalculus

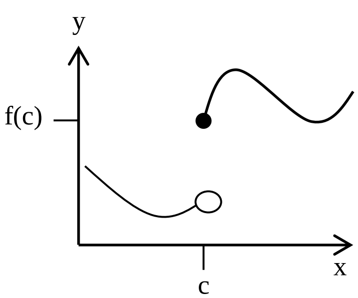
A function is said to be continuous at c if its graph passes through the point at $x = c$ without a "hole" or a "jump"



Continuous at c



Discontinuous at c



Discontinuous at c

Continuity from Calculus

A function f is continuous at c if the following three conditions hold:

1. $f(c)$ is defined
2. $\lim\limits_{x \to c} f(x)$ exists
3. $\lim\limits_{x \to c} f(x) = f(c)$

f is *discontinuous* at c if one or more of these conditions fails to be true.

Which Functions Are Continuous?

If functions f and g are continuous at c, then the following are also continuous at c:

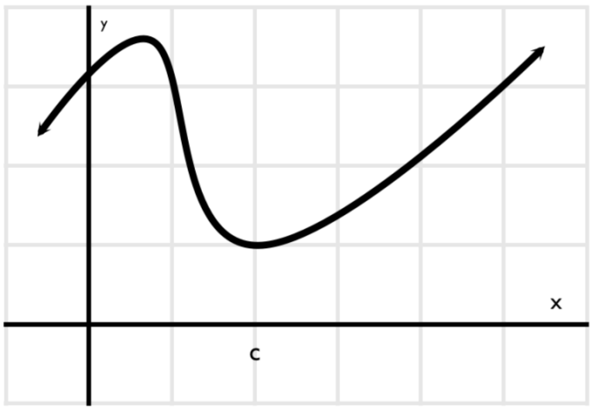
1. $f \pm g$
2. $a \bullet f$ [for any constant a]
3. $f \bullet g$
4. f/g [if $g(c) \neq 0$]
5. $f(g(x))$ [for f continuous at $g(c)$]

All polynomial functions are continuous. Rational functions are not continuous when the denominator = 0 (vertical asymptote). Piece-wise functions have the potential to be continuous or not.

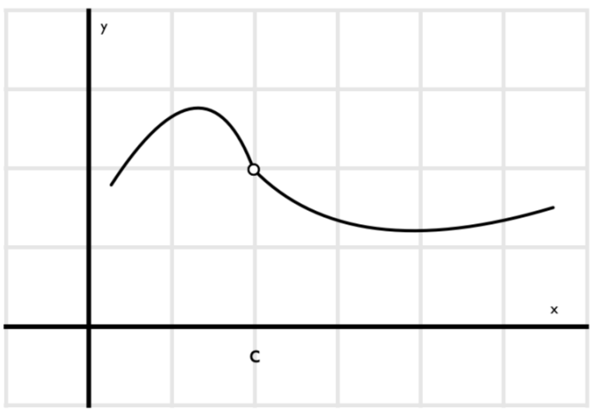
Limits & Continuity
1.4 – Continuity

Ex A: Determine if each function is continuous. If discontinuous, state why.

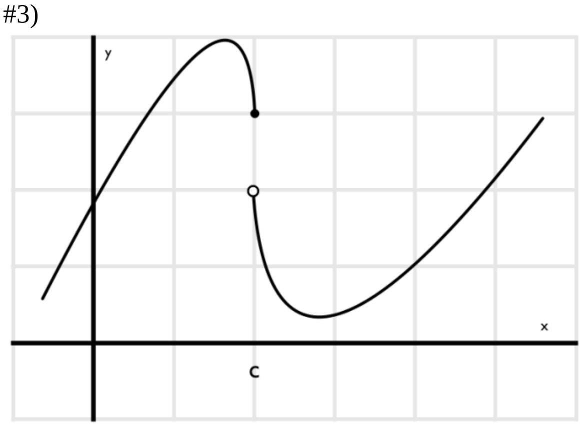
#1)



#2)



#3)



Ex B: Determine if each function is continuous. If discontinuous, state where it is discontinuous and why.

#1) $f(x) = x^3 - 3x^2 - x + 3$

#2) $f(x) = \frac{1}{(x-1)^2}$

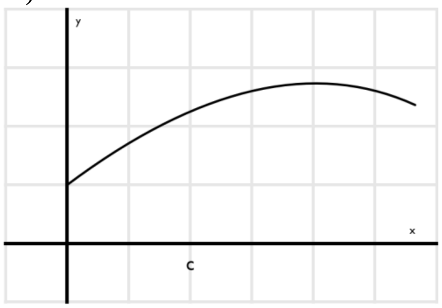
#3) $f(x) = \begin{cases} 2x+1 & if\ x < 2 \\ -2x+9 & if\ x \geq 2 \end{cases}$

#4) $f(x) = \begin{cases} x^2+1 & if\ x < 4 \\ 5x-1 & if\ x \geq 4 \end{cases}$

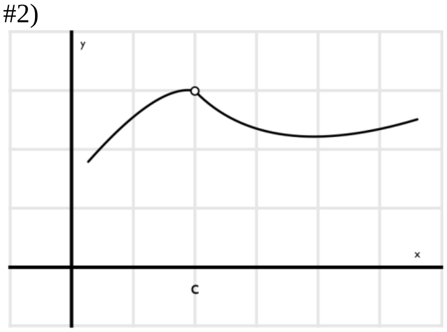
Limits & Continuity
1.4A – Continuity

A: Determine whether each function is continuous at c. If discontinuous, state why.

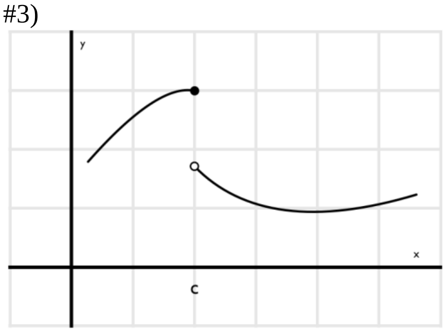
#1)



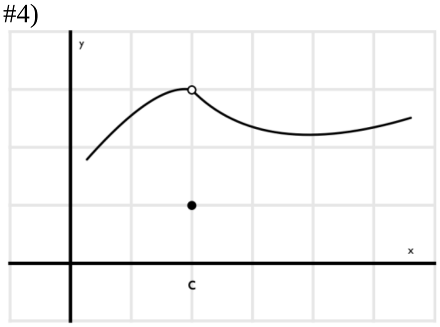
#2)



#3)



#4)



B: Determine whether each function is continuous. If discontinuous, state where it is discontinuous. (You've graphed some of these functions on previous homework.)

#5) $f(x) = 6x + 8$

#6) $f(x) = \frac{x+2}{x-2}$

#7) $f(x) = \frac{1}{x^2+29x+28}$

#8) $f(x) = \begin{cases} x & if\ x < 0 \\ x - 6 & if\ x \geq 0 \end{cases}$

B: *…continued*

#9) $f(x) = \begin{cases} 2x+1 & if\ x<2 \\ -2x-1 & if\ x \geq 2 \end{cases}$

#10) $f(x) = \begin{cases} \frac{1}{3}x+5 & if\ x<9 \\ x-1 & if\ x \geq 9 \end{cases}$

#11) $f(x) = \begin{cases} 2x & if\ x<3 \\ -2x+12 & if\ x \geq 3 \end{cases}$

C: Decide if each statement is true or false. If false give a counterexample. (A counterexample makes the hypothesis true and the conclusion false.)

#12) If $\lim_{x\to 5} f(x) = 10$, then $\lim_{x\to 5^+} f(x) = 10$

#13) If $\lim_{x\to 5^+} f(x) = 10$, then $\lim_{x\to 5} f(x) = 10$

#14) If $f(1) = 7$, then $\lim_{x\to 1} f(x) = 7$

#15) If $f(-4)$ is not defined, then $\lim_{x\to -4} f(x)$ does not exist.