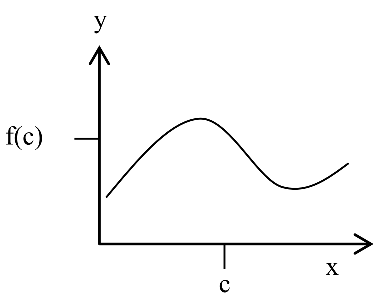
Limits & Continuity

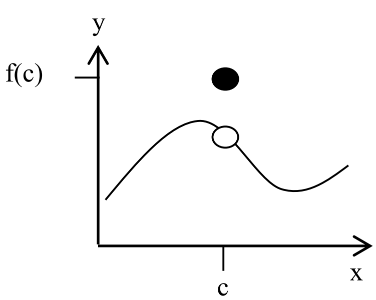
1.4 – Continuity

Continuity from PreCalculus

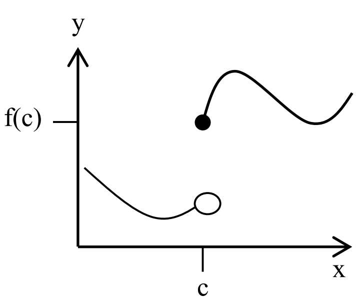
A function is said to be continuous at c if its graph passes through the point at $x = c$ without a "hole" or a "jump"



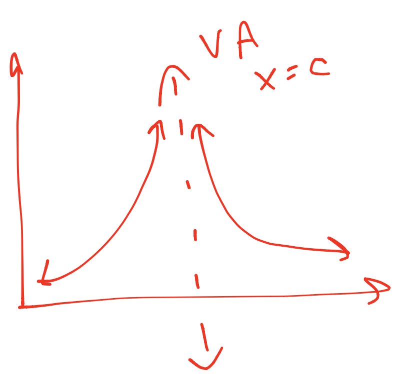
Continuous at c



Discontinuous at c



Discontinuous at c



Continuity from Calculus

A function f is continuous at c if the following three conditions hold:

1. $f(c)$ is defined
2. $\lim\limits_{x\to c} f(x)$ exists
3. $\lim\limits_{x\to c} f(x) = f(c)$

f is *discontinuous* at c if one or more of these conditions fails to be true.

Which Functions Are Continuous?

If functions f and g are continuous at c, then the following are also continuous at c:

1. $f \pm g$
2. $a \bullet f$ [for any constant a]
3. $f \bullet g$
4. f/g [if $g(c) \neq 0$]
5. $f(g(x))$ [for f continuous at $g(c)$]

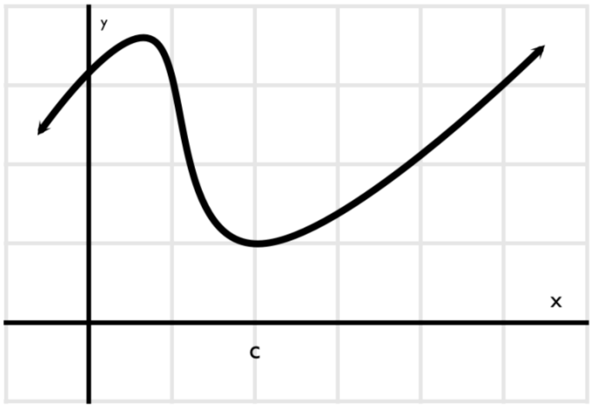
All polynomial functions are continuous. Rational functions are not continuous when the denominator = 0 (vertical asymptote). Piece-wise functions have the potential to be continuous or not.

Limits & Continuity

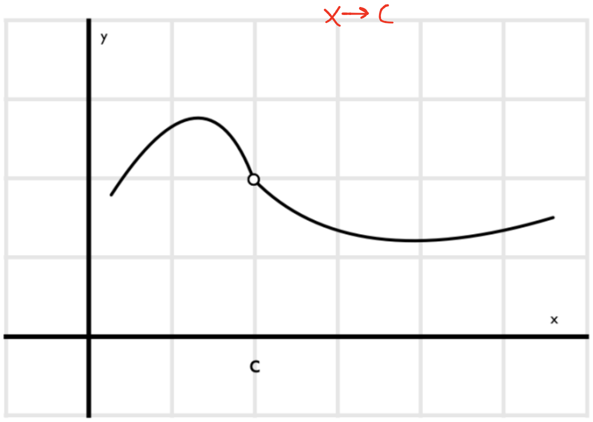
1.4 – Continuity

Ex A: Determine if each function is continuous. If discontinuous, state why.

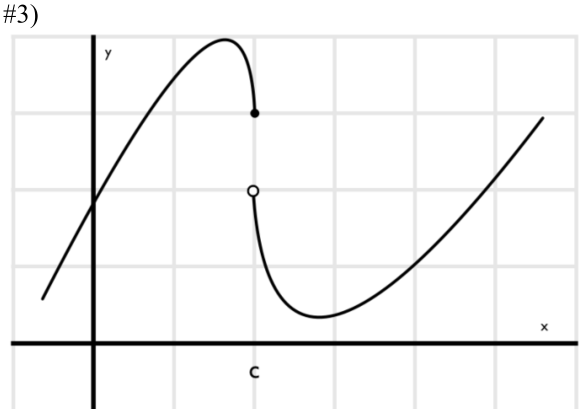
#1) Continuous



#2) Discontinuous, $\lim_{x \to c} f(x) \neq f(c)$



#3)



Discontinuous, $\lim_{x \to c} f(x) = \text{dne}$.

Ex B: Determine if each function is continuous. If discontinuous, state where it is discontinuous and why.

#1) $f(x) = x^3 - 3x^2 - x + 3$

Continuous

#2) $f(x) = \frac{1}{(x-1)^2}$

VA

$(x-1)^2 = 0$

$x - 1 = 0$

$x = 1$

Discontinuous @ $x = 1$

The $\lim_{x \to 1} f(x) = \text{dne}$

#3) $f(x) = \begin{cases} 2x + 1 & if\ x < 2 \\ -2x + 9 & if\ x \geq 2 \end{cases}$

$f(x) = 2x + 1$

$f(2) = 2(2) + 1$

$= 4 + 1$

$f(2) = 5$

$f(x) = -2x + 9$

$f(2) = -2(2) + 9$

$f(2) = -4 + 9$

$f(2) = 5$

Continuous

#4) $f(x) = \begin{cases} x^2 + 1 & if\ x < 4 \\ 5x - 1 & if\ x \geq 4 \end{cases}$

$f(x) = x^2 + 1$

$f(4) = (4)^2 + 1$

$f(4) = 16 + 1$

$f(4) = 17$

$f(x) = 5x - 1$

$f(4) = 5(4) - 1$

$f(4) = 20 - 1$

$f(4) = 19$

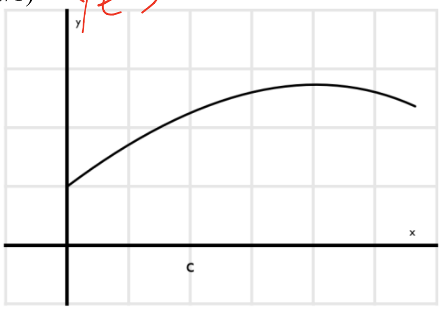
Discontinuous @ $x = 4$

$\lim_{x \to 4} f(x) = \text{dne}$

Limits & Continuity
1.4A – Continuity

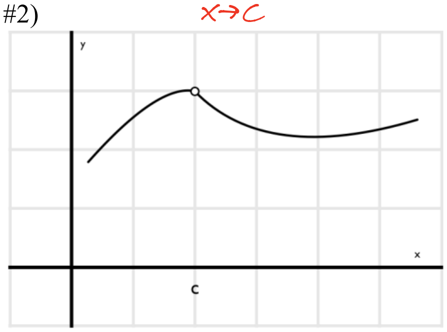
A: Determine whether each function is continuous at c. If discontinuous, state why.

#1) Yes



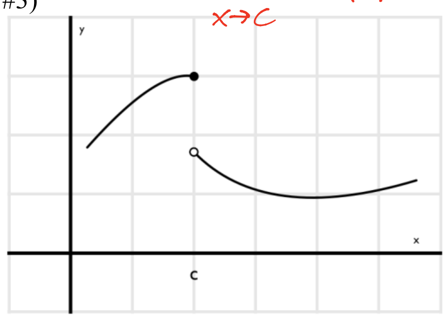
No, $\lim_{x\to c} f(x) \neq f(c)$

#2)



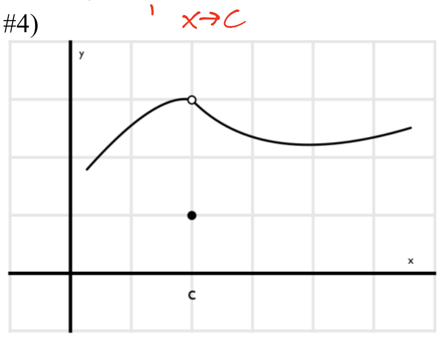
No $\lim_{x\to c} f(x) \neq$ d.n.e.

#3)



NO, $\lim_{x\to c} f(x) \neq f(c)$

#4)



B: Determine whether each function is continuous. If discontinuous, state where it is discontinuous. (You've graphed some of these functions on previous homework.)

#5) $f(x) = 6x + 8$

Continuous

#6) $f(x) = \frac{x+2}{x-2}$

VA

$x - 2 = 0$

$x = 2$

Discontinuous @ $x = 2$

#7) $f(x) = \frac{1}{x^2+29x+28}$

VA

$x^2 + 29x + 28 = 0$

$(x+1)(x+28) = 0$

$x + 1 = 0$ | $x + 28 = 0$

$x = -1$ | $x = -28$

Discontinuous @ $x = -28$ and -1

#8) $f(x) = \begin{cases} x & if\ x < 0 \\ x - 6 & if\ x \geq 0 \end{cases}$

$f(x) = x$

$f(0) = 0$

$f(x) = x - 6$

$f(0) = (0) - 6$

$f(0) = -6$

Discontinuous @ $x = 0$

B: *...continued*

#9) $f(x) = \begin{cases} 2x+1 & if\ x < 2 \\ -2x-1 & if\ x \geq 2 \end{cases}$

$f(x) = 2x+1$

$\boxed{\begin{aligned} f(2) &= 2(2)+1 \\ &= 4+1 \\ f(2) &= 5 \end{aligned}}$

$f(x) = -2x-1$

$\boxed{\begin{aligned} f(2) &= -2(2)-1 \\ &= -4-1 \\ f(2) &= -5 \end{aligned}}$

Discontinuous @ $x = 2$

#10) $f(x) = \begin{cases} \frac{1}{3}x+5 & if\ x < 9 \\ x-1 & if\ x \geq 9 \end{cases}$

$f(x) = \frac{1}{3}x+5$

$\boxed{\begin{aligned} f(9) &= \tfrac{1}{3}(9)+5 \\ &= 3+5 \\ f(9) &= 8 \end{aligned}}$

$f(x) = x-1$

$\boxed{\begin{aligned} f(9) &= (9)-1 \\ f(9) &= 8 \end{aligned}}$

Continuous

#11) $f(x) = \begin{cases} 2x & if\ x < 3 \\ -2x+12 & if\ x \geq 3 \end{cases}$

$f(x) = 2x$

$\boxed{\begin{aligned} f(3) &= 2(3) \\ f(3) &= 6 \end{aligned}}$

$f(x) = -2x+12$

$\boxed{\begin{aligned} f(3) &= -2(3)+12 \\ &= -6+12 \\ f(3) &= 6 \end{aligned}}$

Continuous

C: Decide if each statement is true or false. If false give a counterexample. (A counterexample makes the hypothesis true and the conclusion false.)

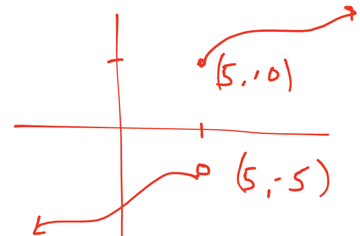
#12) If $\lim_{x\to5} f(x) = 10$, then $\lim_{x\to5^+} f(x) = 10$

True

#13) If $\lim_{x\to5^+} f(x) = 10$, then $\lim_{x\to5} f(x) = 10$

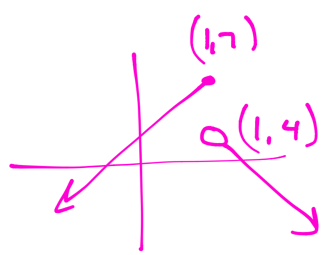
False

Counterexample



#14) If $f(1) = 7$, then $\lim_{x\to1} f(x) = 7$

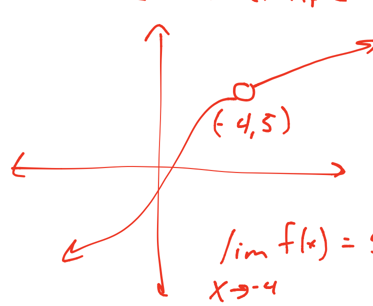
FALSE



#15) If $f(-4)$ is not defined, then $\lim_{x\to-4} f(x)$ does not exist.

False

Counterexample



$\lim_{x\to-4} f(x) = 5$