Advanced Derivative Rules
4.1 – The Chain Rule

The Chain Rule
If h and k are functions of x, then

$$\big[h\big(k(x)\big)\big]' = h'\big(k(x)\big) \cdot k'(x)$$

Prologue

In order to use the Chain Rule, we must first have a composite function. That is, we must have a function inside a function. Let's review how to compose two functions and then how to decompose two functions.

Compose the functions by finding $h(k(x))$.

#1) $h(x) = x^3$
$k(x) = x + 1$

#2) $h(x) = x^2$
$k(x) = \frac{1}{x}$

Decompose the functions by finding functions $h(x)$ and $k(x)$ such that the following expression is the composition $h(k(x))$.

#1) $(x^3 + 9)^{10}$

#2) $\sqrt{x^3 + x^2 - 1}$

Ex A: Find each derivative

#1) $f(x) = (x^2 - 4x + 1)^5$

#2) $y = (x^4 + x^2 + 8)^6$

#3) $g(x) = (5x^2 + x)^{10}$

#4) $y = \left(\frac{1}{x^2 - 1}\right)^3$

#5) $h(x) = \sqrt{x^2 - 10x + 5}$

Giant Ball of Oil

A giant ball of oil was dropped from the 10th floor of Kramerica Industries. Upon impacting the ground, the oil began to expand on the ground in a circular fashion. After *t* days of impacting the ground, the radius of the oil slick is $r(t) = \sqrt{8t + 1}$ feet. How fast is the radius of the oil slick expanding after 1 day?