

Advanced Derivative Rules

4.2 – Derivatives & Trigonometry

Derivatives of the Six Trigonometric Functions

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

Basic Identities

Reciprocal Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$$\begin{aligned} \sin(x) &= \frac{1}{\csc(x)} & \csc(x) &= \frac{1}{\sin(x)} \\ \cos(x) &= \frac{1}{\sec(x)} & \sec(x) &= \frac{1}{\cos(x)} \\ \tan(x) &= \frac{1}{\cot(x)} & \cot(x) &= \frac{1}{\tan(x)} \end{aligned}$$

Quotient Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Pythagorean Identities: The following trig identities hold for all values of A except those for which any function is undefined.

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$1 + \cot^2(x) = \csc^2(x)$$

Even & Odd Trig Functions

Recall that an even function is said to be even if its symmetric with respect to the y-axis, $f(x) = f(-x)$.

$$\cos(-x) = \cos(x)$$

Recall that an odd function is said to be odd if it is symmetric with respect to the origin, $-f(x) = f(-x)$.

$$\sin(-x) = -\sin(x)$$

$$\tan(-x) = -\tan(x)$$

Ex A: Differentiating using the Chain Rule.

#1) $\frac{d}{dx} [\sin^4(x)]$

#2) $\frac{d}{dx} [\sin(x^4)]$

#3) $\frac{d}{dx} [\cos^4(x)]$

#4) $\frac{d}{dx} [\cos(x^4)]$

Ex B: Differentiating using the Product Rule.

#5) $\frac{d}{dx} (x^4 \sin(x))$

Advanced Derivative Rules

4.2 – Derivatives & Trigonometry

#6) If $f(x) = \sin(x) \sec(x)$, find $f'(x)$.

#8) Use the identity $\cot(x) = \frac{\cos(x)}{\sin(x)}$ to prove
 $\frac{d}{dx} \cot(x) = -\csc^2(x)$

Ex C: Differentiating using the Quotient Rule.

#7) If $y = \frac{x^2}{\sin(x)}$, find y' .

#9) Differentiating using Regret, Sadness and a heavy dose of Eraser.

If $y = \frac{\tan(x)}{\sin(x)}$, find y' .