# Limits and Continuity
## Exam Review 1

Find the following limits *without* using a graphing calculator or making tables.

#1) $\lim_{x\to 5} \frac{x^2-15}{2x-5} = \frac{(5)^2-15}{2(5)-5}$

$= \frac{25-15}{10-5}$

$= \frac{10}{5}$

$\lim_{x\to 5} \frac{x^2-15}{2x-5} = 2$

#2) $\lim_{h\to 0} \frac{10x^2h-4xh^2}{h} = \lim_{h\to 0} \frac{h(10x^2-4xh)}{h}$

$= \lim_{h\to 0} (10x^2-4xh)$

$= 10x^2 - 4x(0)$

$\lim_{h\to 0} \frac{10x^2h-4xh^2}{h} = 10x^2$

Answer each question concerning piecewise functions.

#3) $f(x) = \begin{cases} -\frac{1}{5}x+3, & if\ x<5 \\ 2x-8, & if\ x\geq 5 \end{cases}$

a. $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} (-\frac{1}{5}x+3)$

$= -\frac{1}{5}(5)+3$

$= -1+3$

$= 2$

b. $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} (2x-8)$

$= 2(5)-8$

$= 10-8$

$= 2$

c. $\lim_{x\to 5} f(x) = 2$

#4) $f(x) = \begin{cases} 4x+5, & if\ x<1 \\ -2x+7, & if\ x\geq 1 \end{cases}$

a. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (4x+5)$

$= 4(1)+5$

$= 4+5$

$= 9$

b. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (-2x+7)$

$= -2(1)+7$

$= -2+7$

$= 5$

c. $\lim_{x\to 1} f(x) =$ dne.

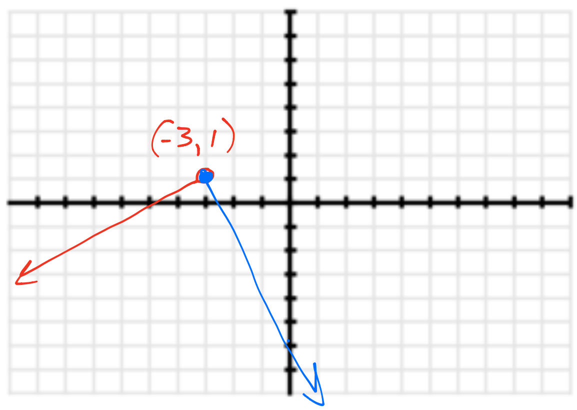
#5) For the following piecewise function:

$$f(x) = \begin{cases} \frac{1}{3}x + 2, & if\ x < -3 \\ -2x - 5, & if\ x \geq -3 \end{cases}$$

a. Draw its graph

| x | $\frac{1}{3}x+2$ |
|---|---|
| -3 | -1 |
| -6 | 0 |

| x | -2x -5 |
|---|---|
| -3 | 1 |
| 0 | - 5 |




b. Find the limits as x approaches -3 from the left.

$\lim_{x \to -3^-} f(x) = 1$

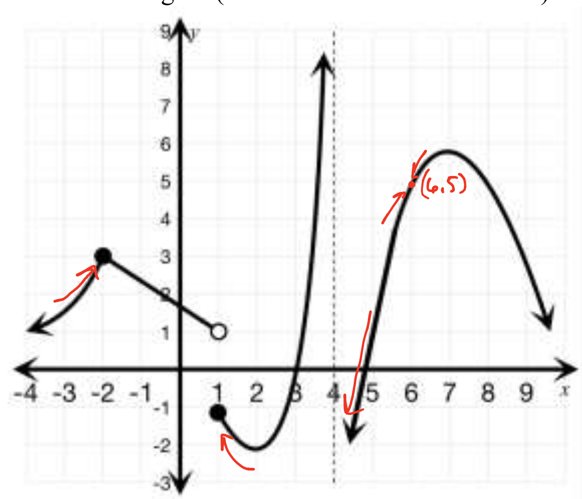
c. Find the limits as x approaches -3 from the right.

$\lim_{x \to -3^+} f(x) = 1$

d. Is it continuous at x = -3? If not, why?

The graph is continuous

#6) Find each limit. Assume that each limit that does exist is an integer. (There is no work to be shown)




a. $\lim_{x \to 1^+} f(x) =$

-1

b. $\lim_{x \to -2^-} f(x) =$

3

c. $\lim_{x \to 4^+} f(x) =$

$-\infty$, d.n.e.

d. $\lim_{x \to 6} f(x) =$

5

#7) Find the equation for the tangent line to the curve $f(x) = 3x^2 - 3x + 1$ at $x = 0$. Write your equation in slope-intercept form. (You must use the definition of derivative to find the slope.)

Point @ $x = 0$

$$f(0) = 3(0)^2 - 3(0) + 1$$
$$f(0) = 1$$
$$(0, 1)$$

Slope

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{[3(x+h)^2 - 3(x+h) + 1] - [3x^2 - 3x + 1]}{h}$$
$$= \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x - 3h + 1 - 3x^2 + 3x - 1}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6hx + 3h^2 \cancel{- 3x} - 3h \cancel{+ 1} \cancel{- 3x^2} \cancel{+ 3x} \cancel{- 1}}{h}$$
$$= \lim_{h \to 0} \frac{6hx + 3h^2 - 3h}{h}$$
$$= \lim_{h \to 0} \frac{h(6x + 3h - 3)}{h}$$
$$= \lim_{h \to 0} (6x + 3h - 3)$$
$$= 6x + 3(0) - 3$$
$$= 6x - 3$$

Slope @ $x = 0$

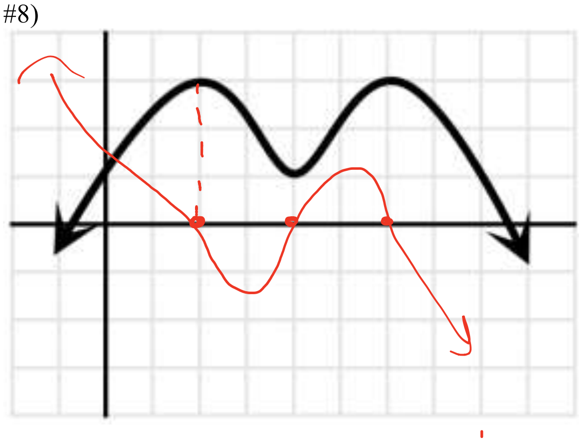
$$f'(0) = 6(0) - 3$$
$$f'(0) = -3$$
$$m = -3$$

Point-slope form

$$y - y_1 = m(x - x_1)$$
$$y - (1) = -3(x - (0))$$
$$y - 1 = -3x$$
$$y = -3x + 1$$

Given the graph of a function, sketch in the graph of its derivative function.

#8)



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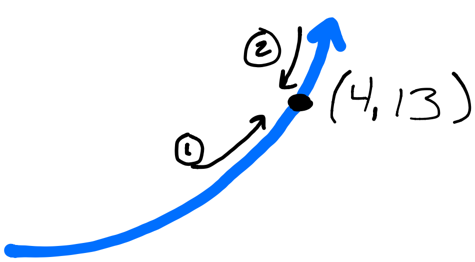
#9) Often times, problems will ask for the derivative without using the word "derivative". We have learned two interpretations of a derivative. What are these two interpretations?

Derivative means slope of a tangent line.

It also means instantaneous rate of change.

#10) $\lim_{x \to 4}(x^2 - 3) = 13$ is read the "limit of $x^2 - 3$, as x approaches 4, is equal to 13." Use sentences and graphs to illustrate the meaning of said statement.

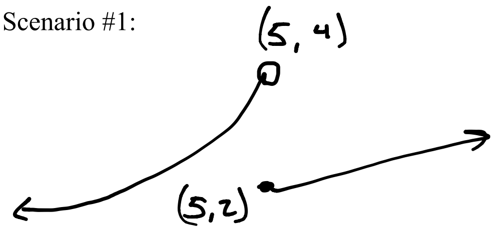



① As x gets closer and closer to 4 from the left, y gets closer and closer to 13.

② As x approaches 4 from the right, y approaches 13.

#11) Give 2 specific scenarios of when a limit would not exist and <u>explain why</u>. You *may* use graphs to illustrate your point.

Scenario #1:



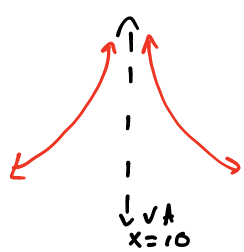

The limit would not exist at $x = 5$.

As $x \to 5^-$ the limit is 4.

As $x \to 5^+$ the limit is 2.

The two-sided limit does not exist because the left and right limits do not agree

Scenario #2:




The limit does not exist @ $x = 10$.
As $x \to 10$ from the left or the right, the y-value approaches infinity.

For a limit to exist, it must approach a single number.
($\infty$ is not a number)