Derivatives Basics
Exam Review 2

#1) Find the equation for the tangent line to the curve $f(x) = 3x^2 - 12x + 9$ at $x = 1$. Write the answer in slope-intercept form.

Point	Slope
$f(1) = 3(1)^2 - 12(1) + 9$	$f'(x) = 6x - 12$
$= 3(1) - 12 + 9$	$f'(1) = 6(1) - 12$
$= 3 - 3$	$= 6 - 12$
$f(1) = 0$	$f'(1) = -6$

Point-Slope

$$y - y_1 = m(x - x_1)$$
$$y - (0) = -6[x - (1)]$$
$$y = -6x + 6$$

#2) If $g(p) = 2\sqrt[6]{p^5} - \frac{12}{\sqrt[6]{p}}$ find $\frac{dg}{dp}$

$$g(p) = 2p^{5/6} - 12p^{-1/6}$$
$$\frac{dg}{dp} = \frac{10}{6}p^{-1/6} + 2p^{-7/6}$$
$$\frac{dg}{dp} = \frac{5}{3\sqrt[6]{p}} + \frac{2}{\sqrt[6]{p^7}}$$

#3) If $f(x) = (x^3 + 2x + 1)$ find $\left.\frac{df}{dx}\right|_{x=-3}$

$$f'(x) = 3x^2 + 2$$
$$f'(-3) = 3(-3)^2 + 2$$
$$f'(-3) = 3(9) + 2$$
$$= 27 + 2$$
$$f'(-3) = 29$$

#4) $[\ln(12x^3 - 3x^2)]'$

$$= \frac{(12x^3 - 3x^2)'}{12x^3 - 3x^2}$$
$$= \frac{36x^2 - 6x}{12x^3 - 3x^2}$$
$$= \frac{{}^{2}\cancel{6x}(6x-1)}{\cancel{3x^2}(4x-1)}$$
$$= \frac{12x - 2}{4x^2 - x}$$

#5) A cooties outbreak begins in a girl's playhouse. The total number of people infected with cooties t days after the first case is $Z(t) = 15t^2 - t^3$ (for $0 \leq t \leq 15$).

t = days	$Z(t)$ = people cooties	$\frac{dZ}{dt} = \frac{\text{people}}{\text{day}}$

Find the instantaneous rate of change on day 3 and interpret your answer.

$$Z'(t) = 30t - 3t^2$$
$$Z'(3) = 30(3) - 3(3)^2$$
$$= 90 - 3(9)$$
$$= 90 - 27$$
$$Z'(3) = 63 \ \frac{\text{cooties}}{\text{day}}$$

Instantaneous rate of change on day 3: 63 people/day

Interpretation:

Three days after a cooties outbreak, the number of infected is increasing by 63 people per day.

#6) After working out to get shredded, George began to stink. His stench was so thick and smelly that people began to pass out. *X* minutes after working out, the number of people who passed out due to George's stench was
$F(x) = 0.1x^2 + 3x$. (for $5 \le x \le 20$).

$x = \text{min}$ | $F = \text{People}$ | $\frac{dF}{dx} = \text{People/min}$

Find $F'(x)$, $F'(10)$ and interpret your answer.

$$F'(x) = 0.2x + 3$$

$$F'(10) = 0.2(10) + 3$$
$$= 2 + 3$$
$$F'(10) = 5$$

$F'(x) =$ $0.2x + 3$

$F'(10) =$ 5 people/min

Interpret your answer:

TEN minutes after working out, the number of people who are passing out is increasing by 5 people per minute.

#7) $\frac{d}{dx} e^{8x^2+4x-1}$

$$= \frac{d}{dx}(8x^2+4x-1)e^{8x^2+4x+1}$$
$$= (16x+4)e^{8x^2+4x+1}$$

#8) $f(x) = \frac{x^3-1}{x^2+1}$; find $f'(x)$

$$f'(x) = \frac{(x^3-1)'(x^2+1) - (x^3-1)(x^2+1)'}{(x^2+1)^2}$$
$$= \frac{3x^2(x^2+1) - (x^3-1)(2x)}{(x^2+1)^2}$$
$$= \frac{3x^4 + 3x^2 - 2x^4 + 2x}{(x^2+1)^2}$$
$$f'(x) = \frac{x^4 + 3x^2 + 2x}{(x^2+1)^2}$$

#9) Differentiate $f(x) = (\ln(x) + e^x - x^2 + 1)\left(\frac{1}{5}x^5 + x^4 - \frac{3}{2}x^2 + 17x\right)$

$$f'(x) = [\ln(x) + e^x - x^2 + 1]'\left[\tfrac{1}{5}x^5 + x^4 - \tfrac{3}{2}x^2 + 17x\right] + [\ln(x) + e^x - x^2 + 1]\left[\tfrac{1}{5}x^5 + x^4 - \tfrac{3}{2}x^2 + 17x\right]'$$

$$f'(x) = \left[\tfrac{1}{x} + e^x - 2x\right]\left[\tfrac{1}{5}x^5 + x^4 - \tfrac{3}{2}x^2 + 17x\right] + [\ln(x) + e^x - x^2 + 1]\left[x^4 + 4x^3 - 3x + 17\right]$$

#10) Why is the derivative referred to as an "instantaneous" rate of change rather than just an "average" rate of change?

An average rate of change is just the slope formula. It is how you calculate the slope of a secant line which requires two points, or two moments in time. Because you are measuring at two points, you are finding the average change that happens between the points.

The derivative is the slope formula with "the limit as h approaches 0" in front of it. By adding the limit as h approaches 0 to the slope formula, the distance between the two points needed to find the slope shrinks down to 0, giving the instantaneous rate of change at one moment in time.