Derivative Applications
Exam Review 3

#1) If $f(x) = (5x^2 + 3x - 1)(x^2 + 2x - 1)$, find the first derivative.

$f'(x) = (5x^2+3x-1)'(x^2+2x-1) + (5x^2+3x-1)(x^2+2x-1)'$

$= (10x+3)(x^2+2x-1) + (5x^2+3x-1)(2x+2)$

#2) If $f(x) = \frac{x^2}{x-1}$, find the first derivative.

$f'(x) = \frac{x'(x-1) - x(x-1)'}{(x-1)^2}$

$f'(x) = \frac{(1)(x-1) - x(1)}{(x-1)^2}$

$f'(x) = \frac{x-1-x}{(x-1)^2}$

$f'(x) = \frac{-1}{(x-1)^2}$

$f''(x) = \frac{(-1)'(x-1)^2 - (-1)(x^2-2x+1)'}{[(x-1)^2]^2}$

$= \frac{0(x-1)^2 + 1(2x-2)}{(x-1)^4}$

$= \frac{0 + 2x - 2}{(x-1)^4}$

$= \frac{2(x-1)}{(x-1)^4}$

$f''(x) = \frac{2}{(x-1)^3}$

#3) $\frac{d^2}{dr^2}(\pi r^5)|_{r=3}$ (write your answer in terms of π)

$\frac{d}{dr}(\pi r^5) = 5\pi r^4$

$\frac{d^2}{dr^2}(\pi r^5) = 20\pi r^3$

$\frac{d^2}{dr^2}(\pi r^5)\Big|_{r=3} = 20\pi(3)^3$

$= 20\pi(27)$

$\frac{d^2}{dr^2}(\pi r^5)\Big|_{r=3} = 540\pi$

#4) If $f(r) = r^5 + r^4$ find $f''(1)$

$f'(r) = 5r^4 + 4r^3$

$f''(r) = 20r^3 + 12r^2$

$f''(1) = 20(1)^3 + 12(1)^2$

$= 20(1) + 12(1)$

$= 20 + 12$

$f''(1) = 32$

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Lint 4 U

#5) Lint 4 U specializes in selling lint to customers whose pockets are unable to produce their own lint. Lint 4 U's profit function is $P(x) = 20x^2 - 12\sqrt[3]{x}$ dollars, where x is the daily sales of lint.

$-12x^{\frac{1}{3}}$

a. Find the marginal profit function.

$P(x) = 20x^2 - 12x^{\frac{1}{3}}$

$MP(x) = 40x - 4x^{-\frac{2}{3}}$

$MP(x) = 40x - \frac{4}{\sqrt[3]{x^2}}$

b. Find the marginal profit when 8 lints have been sold.

$MP(8) = 40(8) - \frac{4}{\sqrt[3]{8}^2}$

$= 320 - \frac{4}{(2)^2}$

$= 320 - \frac{4}{4}$

$= 320 - 1$

$MP(8) = \$319/\text{lint}$

c. Interpret your previous answer

When 8 lints have been sold, the total profit is increasing by \$319 per lint sold.

or

When 8 lints have been sold, the profit for the next lint sold is \$319

Toe Jam Removal

#6) Jenny is starting a business cleaning toe jam out of the crevices of dirty people's toes. Jenny's company has a cost of \$1 for each jam cleared with fixed costs \$5 per week.

a. Find the cost function.

$C(x) = \$1x + 5$

b. Find the average cost function.

$AC(x) = \frac{C(x)}{x}$

$= \frac{x+5}{x}$

$AC(x) = 1 + 5x^{-1}$

c. Find the marginal average cost function.

$MAC(x) = -5x^{-2}$

$MAC(x) = \frac{-5}{x^2}$

d. Evaluate *MAC(x)* at $x = 10$

$MAC(10) = \frac{-5}{(10)^2}$

$= \frac{-5}{100}$

$MAC(10) = -\$.05$ per jam

e. Interpret your previous answer

When 10 toe jams have been cleaned, the average cost per cleaning is decreasing by 5¢ per jam

Derivative Applications
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Gun Shot

#7) If a bullet from a gun is fired vertically up from the ground, its height t seconds after it is fired will be $s(t) = -16t^2 + 1280t$ feet (neglecting regard for human life, of course.)

a. How long will it take for the bullet to reach the ground? Write your answer as a sentence.

$0 = -16t^2 + 1280t$

$0 = -16t(t - 80)$

$0 = -16t$ or $0 = t - 80$

$0 = t$ or $80 = t$

It will take 80 seconds to hit the ground.

b. Find the impact velocity of the bullet hitting the ground. Write your answer as a sentence.

$v(t) = -32t + 1280$

$v(80) = -32(80) + 1280$

$= -2560 + 1280$

$v(80) = -1280$ ft/sec

Eighty seconds after the bullet is fired, its impact velocity with the ground is 1280 feet per second and is headed down.

Route 66

#8) After driving on Route 66 for t hours Elvis is $s(t) = 15t^2 - 2t^3$ miles due west of his starting point. (for $0 < t < 9$).

a. Find his velocity at 8 hours

$v(8) = 30(8) - 6(8)^2$

$= 240 - 6(64)$

$= 240 - 384$

$v(8) = -144$ mi/hr

b. Interpret your previous answer *as it relates to distance.*

At 8 hours Elvis' velocity is 144 miles per hour east.

c. Find his acceleration at 8 hours.

$a(t) = 30 - 12t$

$a(8) = 30 - 12(8)$

$= 30 - 96$

$a(8) = -66$ mi/hr^2

d. Interpret your previous answer *as it relates to velocity*.

At 8 hours, Elvis' velocity is decreasing by 66 miles

Derivative Applications
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Water

#9) Water is thrown up vertically in the air to a height of $s(t) = 64t - 16t^2$ feet at t seconds.

a. Find the water's distance at 1 second

$s(1) = 64(1) - 16(1)^2$

$= 64 - 16(1)$

$= 64 - 16$

$s(1) = 48$ feet

b. Interpret your previous answer.

One second after the water is thrown into the air, its height is 48 feet.

c. Find the water's velocity at 1 second

$v(t) = 64 - 32t$

$v(1) = 64 - 32(1)$

$v(1) = 64 - 32$

$v(1) = 32$ ft/sec

d. Interpret your previous answer *as it relates to distance.*

One second after the water is thrown into the air, its velocity is 32 feet per second going up.

e. Find the water's acceleration at 1 second

$a(t) = -32$

$a(1) = -32$

f. Interpret your previous answer *as it relates to velocity*.

One second after the water is thrown into the air, its velocity is decreasing by 32 feet per second each second.

Mile High Flush

#10) The contents of a toilet flushed from an airplane will fall a distance of $s(t) = 16t^2$ feet (neglecting decency, of course), where t is the time in seconds after the toilet has been flushed.

a. If it takes 5 seconds to hit the ground, find the impact velocity

$v(t) = 32t$

$v(5) = 32(5)$

$v(5) = 160$ ft/sec

b. Write a sentence for your previous answer.

Five seconds after the toilet is flushed, the velocity of its contents is 160 feet per second traveling down.

c. Find the velocity at 2 seconds

$v(2) = 32(2)$

$v(2) = 64$ ft/sec

d. Write a sentence for your previous answer *as it relates to distance.*

Two seconds after the toilet is flushed, the velocity of its contents is 64 feet per second traveling down.