Derivatives & Their Uses

Exam Review 4

#1) If $h(x) = (8x^7 - 9x + 2)^8$, find $h'(x)$

$$h'(x) = 8(8x^7 - 9x + 2)^7 \cdot (8x^7 - 9x + 2)'$$
$$= 8(8x^7 - 9x + 2)^7 (56x^6 - 9)$$

Answer $h'(x) = 8(8x^7 - 9x + 2)^7 (56x^6 - 9)$

#2) If $f(x) = (x^4 + 4)^7 (x^3 + 9)^5$, find $f'(x)$

$$f'(x) = \left[(x^4+4)^7\right]' (x^3+9)^5 + (x^4+4)^7 \left[(x^3+9)^5\right]'$$
$$= 7(x^4+4)^6 (x^4+4)' (x^3+9)^5 + (x^4+4)^7 (5)(x^3+9)^4 (x^3+9)'$$
$$= 7(x^4+4)^6 (4x^3)(x^3+9)^5 + (x^4+4)^7 (5)(x^3+9)^4 (3x^2)$$
$$= x^2 (x^4+4)^6 (x^3+9)^4 \left[7 \cdot 4x(x^3+9) + 5 \cdot 3(x^4+4)\right]$$
$$= x^2 (x^4+4)^6 (x^3+9)^4 \left[28x^4 + 252x + 15x^4 + 60\right]$$
$$= x^2 (x^4+4)^6 (x^3+9)^4 \left[43x^4 + 252x + 60\right]$$

Answer $f'(x) = x^2 (x^4+4)^6 (x^3+9)^4 \left[43x^4 + 252x + 60\right]$

#3) If $w(x) = \left(\frac{2x-2}{3x+5}\right)^4$ find $w'(x)$.

$$w'(x) = 4\left(\frac{2x-2}{3x+5}\right)^3 \left(\frac{2x-2}{3x+5}\right)'$$

$$= 4\,\frac{(2x-2)^3}{(3x+5)^3}\;\frac{(2x-2)'(3x+5)-(2x-2)(3x+5)'}{(3x+5)^2}$$

$$= \frac{4(2x-2)^3}{(3x+5)^3}\cdot\frac{2(3x+5)-(2x-2)(3)}{(3x+5)^2}$$

$$= \frac{4(2x-2)^3\cdot(6x+10-6x+6)}{(3x+5)^5}$$

$$= \frac{4(2x-2)^3(16)}{(3x+5)^5}$$

Answer $w'(x) = \dfrac{4(2x-2)^3(16)}{(3x+5)^5}$

#4) $\frac{d}{dx}[\sin(\cos(x^2+2x+1))]$

$$= \cos(\cos(x^2+2x+1))\cdot[\cos(x^2+2x+1)]'$$

$$= \cos(\cos(x^2+2x+1))\,(-\sin(x^2+2x+1))\cdot(x^2+2x+1)'$$

$$= \cos(\cos(x^2+2x+1))\,(-\sin(x^2+2x+1))\cdot(2x+2)$$

$$= -(2x+2)\cos(\cos(x^2+2x+1))\sin(x^2+2x+1)$$

Answer $\frac{d}{dx}[\sin(\cos(x^2+2x+1))] = -(2x+2)\cos(\cos(x^2+2x+1))\sin(x^2+2x+1)$

#5) If $k(x) = (\sec(3x^2 - 1))^8$ find $k'(x)$

$$k'(x) = 8\sec^7(3x^2-1)\cdot\left[\sec(3x^2-1)\right]'$$

$$= 8\sec^7(3x^2-1)\cdot \sec(3x^2-1)\tan(3x^2-1)\,(3x^2-1)'$$

$$= 8\sec^7(3x^2-1)\cdot \sec(3x^2-1)\tan(3x^2-1)\,(6x)$$

$$= 48x\sec^8(3x^2-1)\tan(3x^2-1)$$

Answer $k'(x) =$ $48x\sec^8(3x^2-1)\tan(3x^2-1)$

#6) $\frac{d}{dx}\left[\frac{\tan(x)}{\csc^2(x)-\cot^2(x)}\right]$ $= \frac{d}{dx}\left[\frac{\tan(x)}{1}\right]$

$$= \frac{d}{dx}\left[\tan(x)\right]$$

$$= \sec^2(x)$$

Answer $\frac{d}{dx}\left[\frac{\tan(x)}{\csc^2(x)-\cot^2(x)}\right] =$ $\sec^2(x)$

Plumber's Crack In a Can

#7) While bending down to fetch a mishandled Funyun, George had a stupendous idea: He would sell Cans O'Crack to plumbers who were too shy to display their own cracks. His cost function for producing Cans O'Crack is $C(x) = \sqrt{5x^2 + 29}$ dollars, where *x* is the number of cans produced.

a. Find the marginal cost function
b. Evaluate $MC(x)$ when 10 cracked corns have been produced and interpret your answer.

a.
$$C(x) = (5x^2+29)^{\frac{1}{2}}$$
$$MC(x) = \frac{1}{2}(5x^2+29)^{-\frac{1}{2}}(5x^2+29)'$$
$$= \frac{10x}{2\sqrt{5x^2+29}}$$
$$MC(x) = \frac{5x}{\sqrt{5x^2+29}}$$

b.
$$MC(10) = \frac{5(10)}{\sqrt{5(10)^2+29}}$$
$$= \frac{50}{\sqrt{5(100)+29}}$$
$$= \frac{50}{\sqrt{500+29}}$$
$$= \frac{50}{\sqrt{529}}$$
$$= \frac{50}{23}$$
$$MC(10) = \$2.17/\text{can}$$

when 10 cans have been made, the total profit is increasing by \$2.17 per can.

George's Headache

#8) After making \$0.35 from his Cans O'Crack venture, George decided to celebrate. While jumping on his bed (read: frameless mattress), he bumped his head on the skylight (read: whole in roof). George decides to pop an Advil to ease his pain. The strength of George's reaction to a dose of *x* milligrams of Advil is $R(x) = 2x\sqrt{10 - \frac{1}{2}x}$ for $0 \le x \le 20$. If $R'(x)$ is called the sensitivity of the Advil, find George's sensitivity to the Advil for a dose of 10 mg. (Use a sentence answer.)

$$R'(x) = (2x)'(10-\tfrac{1}{2}x)^{\frac{1}{2}} + 2x\left[(10-\tfrac{1}{2}x)^{\frac{1}{2}}\right]'$$
$$= 2(10-\tfrac{1}{2}x)^{\frac{1}{2}} + 2x(\tfrac{1}{2})(10-\tfrac{1}{2}x)^{-\frac{1}{2}}(10-\tfrac{1}{2}x)'$$
$$= 2(10-\tfrac{1}{2}x)^{\frac{1}{2}} + x(10-\tfrac{1}{2}x)^{-\frac{1}{2}}(-\tfrac{1}{2})$$
GCF: $\frac{1}{2}(10-\frac{1}{2}x)^{-\frac{1}{2}}$
$$= \tfrac{1}{2}(10-\tfrac{1}{2}x)^{-\frac{1}{2}}\left[4(10-\tfrac{1}{2}x) - x\right]$$
$$= \frac{1}{2\sqrt{10-\frac{1}{2}x}}\left[40-2x-x\right]$$
$$R'(x) = \frac{40-3x}{2\sqrt{10-\frac{1}{2}x}}$$

$$R'(10) = \frac{40-3(10)}{2\sqrt{10-\frac{1}{2}(10)}}$$
$$= \frac{40-30}{2\sqrt{10-5}}$$
$$= \frac{10}{2\sqrt{5}}$$
$$= \frac{5}{\sqrt{5}}$$
$$R'(10) \approx 2.2$$

George's sensitivity to 10mg of Advil is 2.2