Graphing & Basic Optimization
Exam Review 5

#1) Graph the function by hand $f(x)=\sqrt[3]{(x-1)^2} = (x-1)^{2/3}$

1. CV

$f'(x)=\frac{2}{3}(x-1)^{-1/3}$

$0=\frac{2}{3\sqrt[3]{x-1}}$

ZON: None

ZOD:
$0=3\sqrt[3]{x-1}$
$0=\sqrt[3]{x-1}$
$0=x-1$
$1=x$
CV: $x=1$

2. CP

$f(1)=0$

CP $(1,0)$

3. $f'(x)=\frac{2}{3\sqrt[3]{x-1}}$

$f'(x)=\frac{+}{\sqrt[3]{x-1}}$

$\frac{+}{-}=-$ | $\frac{+}{+}=+$

$f'<0$ | $f'>0$

0 | $f'(1)=$ und | 2

MIN $(1,0)$

4. CV

$f''(x)=-\frac{2}{9}(x-1)^{-4/3}$

$0=\frac{-2}{9(\sqrt[3]{x-1})^4}$

ZON: None

ZOD:
$0=9(\sqrt[3]{x-1})^4$
$0=(\sqrt[3]{x-1})^4$
$0=x-1$
$1=x$
CV: $x=1$

5. CP

$f(1)=0$

$(1,0)$

$f''(x)=\frac{-2}{9(\sqrt[3]{x-1})^4}$

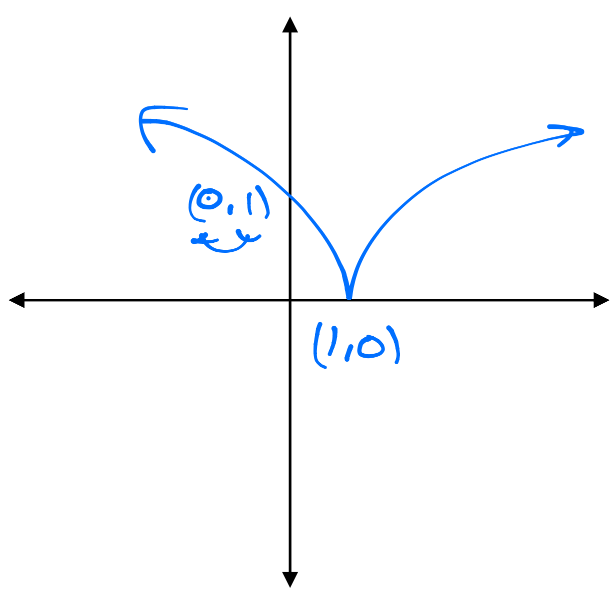
$f''(x)=\frac{-}{+}$

6. $-$ | $-$

$f''<0$ | $f''<0$

$f''(1)=$ und

CC DN | CC DN



7. Y-int

$y=\sqrt[3]{(0-1)^2}$

$y=0$

#2) Find the absolute extreme points of the function on the given interval. $f(x)=\dfrac{x}{x^2+1}$ on [-3, 3]

$$f'(x)=\frac{(x)'(x^2+1)-x(x^2+1)'}{(x^2+1)^2}$$
$$=\frac{(1)(x^2+1)-x(2x)}{(x^2+1)^2}$$
$$=\frac{x^2+1-2x^2}{(x^2+1)^2}$$
$$f'(x)=\frac{1-x^2}{(x^2+1)^2}$$

$0=1-x^2$
$x^2=1$
$x=\pm1$

$0=(x^2+1)^2$
$0=x^2+1$
$-1=x^2$
$\pm\sqrt{-1}=x$
und $=x$

CV: $x=\pm1$

EV: $x=-3, 3$

3

$f(-1)=-\frac{1}{2}$ MIN
$f(1)=\frac{1}{2}$ MAX
$f(-3)=-3/10$
$f(3)=3/10$
CP: $(-1,-\frac{1}{2})$, $(1,\frac{1}{2})$
EP: $(-3,-3/10)$, $(3,\frac{3}{10})$

Absolute Max: $(1,\frac{1}{2})$

Absolute Min: $(-1,-\frac{1}{2})$

Graphing & Basic Optimization
Exam Review 5

#3) A retired potter can produce china pitchers at a cost of $5 each. She estimates her price function to be p = 17 – 0.5x, where p is the price at which exactly x pitchers will be sold per week. Find the number of pitchers that she should produce and the price that she should charge in order to maximize profit. Also find the maximum profit.

$\boxed{C(x) = \$5x}$

$R(x) = p \cdot qty$
$= (17 - 0.5x)\,x$
$R(x) = 17x - 0.5x^2$

$P(x) = R(x) - C(x)$
$= (17x - 0.5x^2) - (5x)$
$P(x) = -0.5x^2 + 12x$

$P'(x) = -1.0x + 12$
$0 = -x + 12$
$x = 12$

$P''(x) = -1$
$P''(12) = -1$, CC DN, MAX

$p = 17 - 0.5x$
$p(12) = 17 - 0.5(12)$
$p(12) = 17 - 6$
$p(12) = 11$

$P(x) = -0.5x^2 + 12x$
$P(12) = -0.5(12)^2 + 12(12)$
$= -0.5(144) + 144$
$= -72 + 144$
$P(12) = 72$

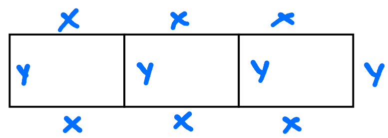
Quantity that should be produced: 12

Price to be sold: $11

Maximum Profit: $72

Sentence Answer: To maximize the profit to $72, she should sell 12 pitchers @ $11 each.

#4) A homeowner wants to enclose three adjacent rectangular pens of equal size, as in the diagram below. What is the largest total area that can be enclosed using exactly 240 feet of fence?



$x =$ horizontal length
$y =$ vertical length

$$P = 6x + 4y$$
$$240 = 6x + 4y$$
$$240 - 6x = 4y$$
$$\frac{240-6x}{4} = y$$
$$60 - \frac{3}{2}x = y$$

$$A = 3xy$$
$$A = 3x\left(60 - \frac{3}{2}x\right)$$
$$A = 180x - \frac{9}{2}x^2$$
$$A' = 180 - 9x$$
$$0 = 180 - 9x$$
$$9x = 180$$
$$x = 20$$

$$A''(x) = -9$$
$A''(20) = -$, CC DN, MAX

$$60 - \frac{3}{2}(20) = y$$
$$60 - 3(10) = y$$
$$60 - 30 = y$$
$$30 = y$$

$$A = 3xy$$
$$A = 3(20)(30)$$
$$A = 3(600)$$
$$A = 1800$$

Sides vertical: 30 ft

Sides horizontal: 20 ft

Total Maximized Area: 1800 ft^2

Sentence Answer: To maximize area to 1800 ft^2, the sides of each enclosure should be 30 ft vertically and 20 ft horizontally.