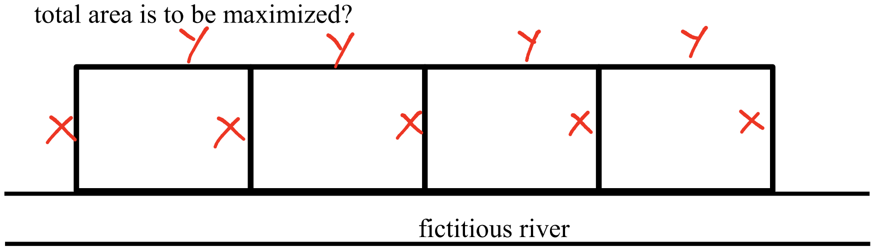
#1) A guy whose name is not George wants to make four identical rectangular enclosures along a fictitious river for his fictitious farm, as in the non-fictitious diagram shown below. If he has 2000 inches of fictitious fence (and if the sides along the fictitious river needs no fence of any kind), what should be the dimensions of each enclosure if the total area is to be maximized?



① $A = x \cdot 4y$

$A = x \cdot 4\left(500 - \frac{5}{4}x\right)$

$A = 2000x - 5x^2$

② $P = 5x + 4y$

$2000 = 5x + 4y$

$2000 - 5x = 4y$

$\frac{2000 - 5x}{4} = y$

$500 - \frac{5}{4}x = y$

③ $A' = 2000 - 10x$

$0 = 2000 - 10x$

$10x = 2000$

$x = 200$

④ $A''(x) = -10$

$A''(200) =$ neg, ccu, MAX

⑤ $500 - \frac{5}{4}(200) = y$

$500 - 250 = y$

$250 = y$

⑥ $A = 4xy$

$= 4(200)(250)$

$A = 200{,}000 \text{ in}^2$

Length of side parallel to river of one enclosure: 250 in

Length of side perpendicular to river of one enclosure: 200 in

Maximum Area: $200{,}000 \text{ in}^2$

#2) George has his very own television network called, The Gnomes of Gruesome George. It basically consists of George driving around the city destroying unsuspecting gnomes. His network, GGG, has 10,000 customers and charges $25 per month. A survey by George's marketing firm (consisting of one-part grandma and two parts Nyquil) indicated that each decrease of $1 in monthly charges will result in 1000 new subscribers. Determine that monthly charges that will result in a maximum monthly revenue.

x = # of $1 decreases

$p(x) = \$25 - x$

$q(x) = 10{,}000 + 1000x$

$R(x) = p(x) \cdot q(x)$

$= (25 - x)(10{,}000 + 1000x)$

$= 250{,}000 + 25{,}000x - 10{,}000x - 1000x^2$

$R(x) = -1000x^2 + 15{,}000x + 250{,}000$

$R'(x) = -2000x + 15{,}000$

$0 = -2000x + 15{,}000$

$2000x = 15{,}000$

$x = 7.5$

$R''(x) = -2000$

$R''(7.5) =$ neg. CCD, MAX

$q(x) = 10{,}000 + 1000x$

$q(7.5) = 10{,}000 + 1000(7.5)$

$= 10{,}000 + 7{,}500$

$q(7.5) = 17{,}500$

$p = 25 - x$

$p = 25 - 7.5$

$p = 17.5$

$R = p \cdot q$

$R = (\$17.50)(17{,}500)$

$R = \$306{,}250$

Number of $1 decreases: 7.5

Quantity: 17,500

Price: $17.50

Maximum Revenue: $306,250

#3) For $x^2 + 4xy + y^2 = 1$, find $\frac{dy}{dx}$ using implicit differentiation.

$$\frac{d}{dx}x^2 + \frac{d}{dx}(4xy) + \frac{d}{dx}y^2 = \frac{d}{dx}1$$

$$2x + \frac{d}{dx}(4x)\cdot y + 4x\frac{d}{dx}y + 2y\frac{dy}{dx} = 0$$

$$2x + 4\cdot y + 4x\cdot\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$4x\cdot\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$$

$$\frac{dy}{dx}(4x+2y) = -2(x+2y)$$

$$\frac{dy}{dx} = \frac{-2(x+2y)}{2(2x+y)}$$

$$\frac{dy}{dx} = \frac{-x-2y}{2x+y}$$

$\frac{dy}{dx} = \frac{-x-2y}{2x+y}$

#4) For $x^{-1} + y^{-1} = 5$, use implicit differentiation to find $\frac{dy}{dx}$ at x = -0.2 and y = 0.1.

$$\frac{d}{dx}x^{-1} + \frac{d}{dx}y^{-1} = \frac{d}{dx}5$$

$$-x^{-2} - y^{-2}\frac{dy}{dx} = 0$$

$$-y^{-2}\frac{dy}{dx} = x^{-2}$$

$$\frac{dy}{dx} = \frac{x^{-2}}{-y^{-2}}$$

$$\frac{dy}{dx} = \frac{y^2}{-x^2}$$

$$\left.\frac{dy}{dx}\right|_{(-0.2,0.1)} = \frac{(0.1)^2}{-(0.2)^2}$$

$$= \frac{.01}{-.04}$$

$$\left.\frac{dy}{dx}\right|_{(-0.2,0.1)} = -\frac{1}{4}$$

$\frac{dy}{dx} = \frac{y^2}{-x^2}$

At (-0.2, 0.1) $\frac{dy}{dx} = -\frac{1}{4}$

#5) George loves gum almost as much as he like making money, so he decides to put his money where his mouth has been. *George's Gently Used Gumballs* will be increasing sales at the rate of 70 gumballs per week. Revenue from the sale of x gumballs is $R(x) = 100x - 0.001x^2$ dollars. Find the rate of change of revenue with respect to time when the weekly sales are 550 gumballs. Give a sentence answer.

x = # of gumball sold
t = time in weeks
R = Revenue

$\frac{dx}{dt} = \frac{70 \text{ gumballs}}{\text{week}}$

FIND $\left.\frac{dR}{dt}\right|_{x=550}$

$$R(x) = 100x - 0.001x^2$$

$$\frac{d}{dt}R = \frac{d}{dt}100x - \frac{d}{dt}0.001x^2$$

$$\frac{dR}{dt} = 100\frac{dx}{dt} - 0.002x\frac{dx}{dt}$$

$$\left.\frac{dR}{dt}\right|_{x=550, \frac{dx}{dt}=70} = 100(70) - 0.002(550)(70)$$

$$= 7{,}000 - 77$$

$$\left.\frac{dR}{dt}\right|_{x=550} = \$6923/\text{week}$$

Sentence answer: when 550 gumballs have been sold, the revenue is increasing by \$6,923 per week.