Derivatives & Their Uses
Exam Review 1

#1) Find the limit by using tables.

$\lim_{x\to 4}\frac{2x+1}{x-4} =$ dne

3.9	3.99	3.999	4.001	4.01	4.1
-88	-898	-8998	9002	902	92

$f(x)\to -\infty$ $\quad \infty \leftarrow f(x)$

$\lim_{x\to 4}\frac{2x+1}{x-4} =$ dne

#2) Find the limit without tables or a graphing calculator.

$\lim_{h\to 0}\frac{x^2h - xh^2 + h^3}{h} =$

$= \lim_{h\to 0}\frac{h(x^2 - xh + h^2)}{h}$

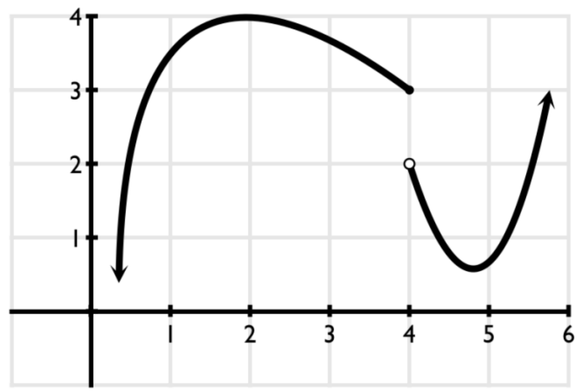
$= \lim_{h\to 0}(x^2 - xh + h^2)$

$= x^2 - x(0) + (0)^2$

$= x^2$

$\lim_{h\to 0}\frac{x^2h - xh^2 + h^3}{h} = x^2$

Use the graph to answer each question.



#3) $\lim_{x\to 4^-} f(x) =$ 3

#4) $\lim_{x\to 4^+} f(x) =$ 2

#5) $\lim_{x\to 4} f(x) =$ dne.

#6) Is the function continuous at $x = 4$? If no, why not?

No $\lim_{x\to 4} f(x) =$ d.n.e.

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#7) Find $f'(x)$ by using the definition of the derivative.

$$f(x) = -5x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[-5(x+h)] - [-5x]}{h}$$

$$= \lim_{h \to 0} \frac{-5x - 5h + 5x}{h}$$

$$= \lim_{h \to 0} \frac{-5h}{h}$$

$$= \lim_{h \to 0} -5$$

$$= -5$$

$f'(x) = -5$

#8) Find the equation for the tangent line to the curve $f(x) = 2x^2 - 6x + 9$ at $x = 3$. Write the answer in slope-intercept form.

Point @ $x=3$

$$f(3) = 2(3)^2 - 6(3) + 9$$
$$= 2(9) - 18 + 9$$
$$= 18 - 18 + 9$$
$$f(3) = 9$$

Slope @ $x=3$

$$f'(x) = 4x - 6$$
$$f'(3) = 4(3) - 6$$
$$= 12 - 6$$
$$f'(3) = 6$$
$$m = 6$$

Point-slope form

$$y - y_1 = m(x - x_1)$$
$$y - (9) = 6(x - (3))$$
$$y - 9 = 6x - 18$$
$$y = 6x - 9$$

Slope-intercept form: $y = 6x - 9$

#9) Ground zero of a zombie virus outbreak began on Heritage Drive Tuscarawas, Ohio. The total number of people infected t days after the first case is $Z(t) = 13t^2 - t^3$ (for $0 \leq t \leq 13$).

Find the instantaneous rate of change on day 5 and interpret your answer.

$Z(t)$ = Zombies

t = days

$Z'(t) = \frac{\text{Zombies}}{\text{day}}$

$$Z'(t) = 26t - 3t^2$$

$$Z'(5) = 26(5) - 3(5)^2$$
$$= 130 - 3(25)$$
$$= 130 - 75$$
$$Z'(5) = 55 \text{ Z/d}$$

Instantaneous rate of change on day 5: 55 Z/d

Interpretation:

On the 5th day of a Zombie outbreak, the number of Zombies is increasing by 55 Zombies per day.

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#10) If $g(p) = 6\sqrt[3]{p^2} - \frac{48}{\sqrt[3]{p}}$ find $\frac{dg}{dp}$

$$g(p) = 6p^{2/3} - 48p^{-1/3}$$

$$\frac{dp}{dg} = \left(\frac{2}{3}\right)\cdot 6p^{-1/3} - \left(-\frac{1}{3}\right)\cdot 48\,p^{-4/3}$$

$$= 4p^{-1/3} + 16p^{-4/3}$$

$$= \frac{4}{\sqrt[3]{p}} + \frac{16}{\sqrt[3]{p^4}}$$

$$\frac{dg}{dp} = \frac{4}{\sqrt[3]{p}} + \frac{16}{\sqrt[3]{p^4}}$$

#11) If $f(x) = x^3 + 2$ find $\left.\frac{df}{dx}\right|_{x=-3}$

$$\left.\frac{df}{dx}\right|_{x=-3} = \left.3x^2\right|_{x=-3}$$

$$= 3(-3)^2$$

$$= 3(9)$$

$$= 27$$

$$\left.\frac{df}{dx}\right|_{x=-3} = 27$$

#12) After eating a putrid sandwich, George began to stink. His stench was so thick and smelly that flies began to hover around him. X minutes after eating the moldy sandwich, the number of flies on George was $F(x) = 0.1x^2 + 3x$. (for $5 \le x \le 20$).

Find $F'(x)$, $F'(10)$ and interpret your answer.

$F(x)$ = flies

x = minutes

$F'(x)$ = flies/min

$$F'(x) = 0.2x + 3$$

$$F'(10) = 0.2(10) + 3$$

$$= 2 + 3$$

$$F'(10) = 5 \text{ F/m}$$

$F'(x) =$ $0.2x + 3$

$F'(10) =$ 5 flies/min

Interpret your answer:

Ten minutes after eating a moldy sandwich the number of flies on George is increasing by 5 flies per minute

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#13) $[\ln(5x^2 - 9x)]'$

$$= \frac{(5x^2-9x)'}{5x^2-9x}$$

$$= \frac{10x-9}{5x^2-9x}$$

$[\ln(5x^2 - 9x)]' = \frac{10x-9}{5x^2-9x}$

#14) $\frac{d}{dx} e^{x^2+x} = (x^2+x)' \cdot e^{x^2+x}$

$$= (2x+1)e^{x^2+x}$$

$\frac{d}{dx} e^{x^2+x} = (2x+1)e^{x^2+x}$

#15) $\frac{d}{dx}[x^5 \sin(x)] = \frac{d}{dx} x^5 \cdot \sin(x) + x^5 \cdot \frac{d}{dx} \sin(x)$

$$= 5x^4 \cdot \sin(x) + x^5 \cos(x)$$

$\frac{d}{dx}[x^5 \sin(x)] = 5x^4 \sin(x) + x^5 \cos(x)$

#16) $\frac{d}{dx}\left[\frac{\tan(x)}{x^2+x+6}\right] = \frac{\frac{d}{dx}\tan(x) \cdot (x^2+x+6) - \tan(x) \frac{d}{dx}(x^2+x+6)}{(x^2+x+6)^2}$

$$= \frac{\sec^2(x) \cdot (x^2+x+6) - \tan(x)(2x+1)}{(x^2+x+6)^2}$$

$\frac{d}{dx}\left[\frac{\tan(x)}{x^2+x+6}\right] = \frac{\sec^2(x) \cdot (x^2+x+6) - \tan(x)(2x+1)}{(x^2+x+6)^2}$

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#17) Name one situation in which a limit would not exist and explain why the limit would not exist.

1) If the left and right limits of "c" are not equal, then the limit does not exit. A limit must approach a single number to exist.

2) If the limit approaches infinity or negative infinity, then the limit does not exist. A limit must approach a single number to exist.

#18) Who are the two people who invented Calculus?

Isaac Newton and Gottfried Leibniz invented calculus.

#19) Why is the derivative referred to as an "instantaneous" rate of change rather than just an "average" rate of change?

An average rate of change is just the slope formula. It is how you calculate the slope of a secant line which requires two points, or two moments in time. Because you are measuring at two points, you are finding the average change that happens between the points.

The derivative is the slope formula with "the limit as h approaches 0" in front of it. By adding the limit as h approaches 0 to the slope formula, the distance between the two points needed to find the slope shrinks down to 0, giving the instantaneous rate of change at one moment in time.

#20) What are three ways to find a limit?

Find by substitution
Find by graphing
Find by tables