Derivatives & Their Uses
Exam Review 2

#1) Find the limits by using any method. Then, circle which method you used.

$$\lim_{x\to 1}\frac{x^2-4}{x^2-4x+4} = \frac{(1)^2-4}{(1)^2-4(1)+4}$$

$$= \frac{1-4}{1-4+4}$$

$$= \frac{-3}{1}$$

$$= -3$$

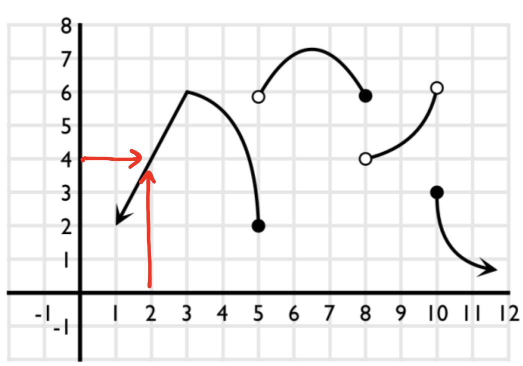
Answer: $\lim_{x\to 1}\frac{x^2-4}{x^2-4x+4} = -3$

Graphing

Table

Substitution

#2) Use the graph to state the limit.



Answer $\lim_{x\to 2} f(x) = 4$

#3) If $f(x) = \frac{9}{x}$ find $f''(x)$.

$f(x) = 9x^{-1}$

$f'(x) = -9x^{-2}$

$f''(x) = 18x^{-3}$

Answer $f''(x) = \frac{18}{x^3}$

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#4) Find the equation for the tangent line to the curve $f(x) = 6x^2 + 8x - 22$ at $x = 2$. Write the answer in slope-intercept form.

Point @ $x=2$

$$f(2) = 6(2)^2 + 8(2) - 22$$
$$= 6(4) + 16 - 22$$
$$= 24 - 6$$
$$f(2) = 18$$
$$(2, 18)$$

Slope @ $x=2$

$$f'(x) = 12x + 8$$
$$f'(2) = 12(2) + 8$$
$$f'(2) = 24 + 8$$
$$f'(2) = 32$$
$$m = 32$$

Point-Slope form

$$y - y_1 = m(x - x_1)$$
$$y - (18) = 32(x - (2))$$
$$y - 18 = 32x - 64$$
$$y = 32x + 46$$

Answer Slope-intercept form $y = 32x + 46$

#5) If $h(x) = (6x^3 - 12x + 19)(x + 8)$ find $\frac{dh}{dx}$.

$$\frac{dh}{dx} = \frac{d}{dx}(6x^3 - 12x + 19)(x + 8) + (6x^3 - 12x + 19)\frac{d}{dx}(x + 8)$$
$$= (18x^2 - 12)(x + 8) + (6x^3 - 12x + 19)(1)$$
$$= 18x^3 + 144x^2 - 12x - 96 + 6x^3 - 12x + 19$$
$$= 24x^3 + 144x^2 - 24x - 77$$

Answer $\frac{dh}{dx} = 24x^3 + 144x^2 - 24x - 77$

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#6) If $h(x) = (8x^7 - 9x + 2)^8$, find $h'(x)$

$$h'(x) = 8(8x^7 - 9x + 2)^7 \cdot (8x^7 - 9x + 2)'$$

$$= 8(8x^7 - 9x + 2)^7 (56x^6 - 9)$$

Answer $h'(x) = 8(8x^7 - 9x + 2)^7 (56x^6 - 9)$

#7) If $f(x) = (x^4 + 4)^7 + (x^3 + 9)^5$, find $f'(x)$

$$f'(x) = 7(x^4+4)^6 (x^4+4)' + 5(x^3+9)^4 (x^3+9)'$$

$$= 7(x^4+4)^6 (4x^3) + 5(x^3+9)^4 (3x^2)$$

$$= 28x^3 (x^4+4)^6 + 15x^2 (x^3+9)^4$$

Answer $f'(x) = 28x^3 (x^4+4)^6 + 15x^2 (x^3+9)^4$

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#8) If $w(x) = \left(\frac{2x-2}{3x+5}\right)^4$ find $w'(x)$.

$$w'(x) = 4\left(\frac{2x-2}{3x+5}\right)^3 \cdot \left(\frac{2x-2}{3x+5}\right)'$$

$$= 4\,\frac{(2x-2)^3}{(3x+5)^3} \cdot \frac{(2x-2)'(3x+5) - (2x-2)(3x+5)'}{(3x+5)^2}$$

$$= \frac{4(2x-2)^3}{(3x+5)^3} \cdot \frac{2(3x+5) - (2x-2)(3)}{(3x+5)^2}$$

$$= \frac{4(2x-2)^3}{(3x+5)^3} \cdot \frac{6x+10-6x+6}{(3x+5)^2}$$

$$= \frac{4(2x-2)^3}{(3x+5)^3} \cdot \frac{16}{(3x+5)^2}$$

Answer $w'(x) = \dfrac{64(2x-2)^3}{(3x+5)^5}$

#9) $\frac{d}{dx}[\sin(x)\csc(x)] = \frac{d}{dx}(1)$

$$= 0$$

Answer $\frac{d}{dx}[\sin(x)\csc(x)] = 0$

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#10) If $k(x) = 5x^3(-6x+1)^8$ find $k'(x)$

$$k'(x) = (5x^3)'(-6x+1)^8 + (5x^3)\left[(-6x+1)^8\right]'$$

$$= 15x^2(-6x+1)^8 + 5x^3(8)(-6x+1)^7(-6x+1)'$$

$$= 15x^2(-6x+1)^8 + 5x^3(8)(-6x+1)^7(-6)$$

$$= 5x^2(-6x+1)^7\left[3(-6x+1) + x(8)(-6)\right]$$

$$= 5x^2(-6x+1)^7\left[-18x+3 - 48x\right]$$

$$= 5x^2(-6x+1)^7\left[-66x+3\right]$$

Answer $k'(x) = 5x^2(-6x+1)^7\left[-66x+3\right]$

#11) $\frac{d}{dx}\left[\frac{\tan(x)}{\csc^2(x)-\cot^2(x)}\right] = \frac{d}{dx}\left[\frac{\tan(x)}{(1+\cot^2(x)) - \cot^2(x)}\right]$

$$= \frac{d}{dx}\left[\frac{\tan(x)}{1}\right]$$

$$= \frac{d}{dx}\left[\tan(x)\right]$$

$$= \sec^2(x)$$

Answer $\frac{d}{dx}\left[\frac{\tan(x)}{\csc^2(x)-\cot^2(x)}\right] = \sec^2(x)$

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#12) After *t* hours a passenger train is $s(t) = 24t^2 - 2t^3$ miles due north of its starting point (for $0 \leq t \leq 12$).

a. Find its velocity at time t = 5 hours. Write your answer as a sentence.

$v(t) = 48t - 6t^2$

$v(5) = 48(5) - 6(5)^2$

$= 240 - 6(25)$

$= 240 - 150$

$v(5) = 90$ mi/h North

Five hours after departing, the passenger train's velocity is 90 miles per hour due north.

b. Find its acceleration at time t = 5 hours. Write your answer as a sentence.

$a(t) = 48 - 12t$

$a(5) = 48 - 12(5)$

$= 48 - 60$

$a(5) = -12$

Five hours after departing, the passenger train's velocity is decreasing by 12 miles per hour each hour.

#13) The population of a city *t* years from now is predicted to be $P(t) = 0.25t^3 + 5t + 200$ people.

a. $P(10) = 500$
Answer Interpretation = Ten years from now the city's population will be 500 people.

b. $P'(10) = 80$
Answer Interpretation =
Ten years from now the city's population will be growing by 80 people per year.

c. $P''(10) = 15$
Answer Interpretation =
Ten years from now the city's population growth rate will be increasing by 15 people per year each year.

#14) A company's cost function is $C(x) = 5x + 100$ dollars, where *x* is the number of units.

a. $\boldsymbol{MAC(20) = -.25}$
Answer Interpretation:
After 20 units have been produced, the average profit per unit is decreasing by 25¢ per unit.