Derivatives & Their Uses
Exam Review 3

#1) Find the critical values of the function from the first and second derivative.
$f(x) = x^3 + 3x^2 - 9x - 11$

CV

$f'(x) = 3x^2 + 6x - 9$

$0 = 3(x^2 + 2x - 3)$

$0 = 3(x+3)(x-1)$

$0 \neq 3$; $x+3=0$, $x=-3$; $x-1=0$, $x=1$

CV: $x = -3, 1$

CV

$f''(x) = 6x+6$

$0 = 6(x+1)$

$0 \neq 6$; $0 = x+1$, $-1 = x$

CV: $x = -1$

#2) Sketch the graph by hand, showing all relative extreme points and y-intercept.
$f(x) = x^4 - 8x^3 + 18x^2$

CV: $x = 0, 3$

$f'(x) = 4x^3 - 24x^2 + 36x$

$0 = 4x(x^2 - 6x + 9)$

$0 = 4x(x-3)^2$

$0 = 4x$, $0 = x$; $0 = (x-3)^2$, $0 = x-3$, $3 = x$

CP: (0,0), (3,27)

$f(0) = 0$

$f(3) = 27$

$f'(x) = 4x(x-3)^2$

(+)(−)(+) | (+)(+)(+) | (+)(+)(+)

$f' < 0$ | $f' > 0$ | $f' > 0$

−1 $f'(0)=0$ 2 $f'(3)=0$ 4

MIN (0,0) (3,27)

y-int

$f(0)=0$

CV: $x = 1, 3$

$f''(x) = 12x^2 - 48x + 36$

$0 = 12(x^2 - 4x + 3)$

$0 = 12(x-3)(x-1)$

$0 \neq 12$; $x-3=0$, $x=3$; $x-1=0$, $x=1$

CP: (1,11), (3,27)

$f(1) = 11$

$f(3) = 27$

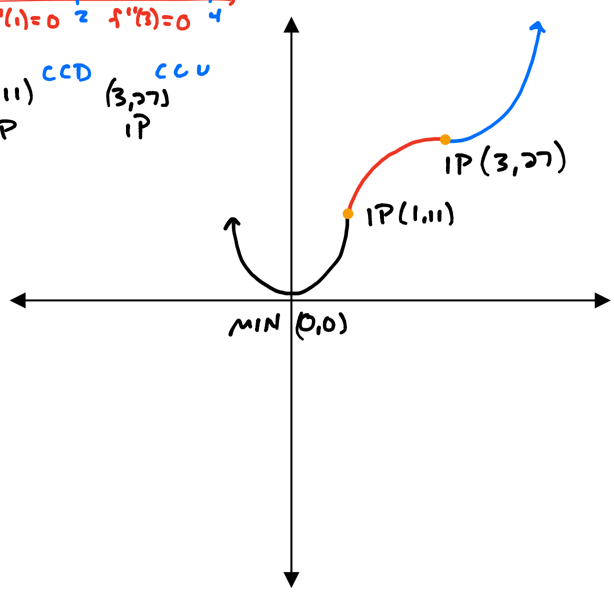
$f''(x) = 12(x-3)(x-1)$

(+)(−)(−) | (+)(+)(−) | (+)(+)(+)

$f' > 0$ | $f' < 0$ | $f' > 0$

−1 $f''(1)=0$ 2 $f''(3)=0$ 4

CCU (1,11) IP CCD (3,27) IP CCU



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Find the absolute extreme points of each function on the given interval.

#3) $f(x) = x^3 + 3x^2 - 4$ on [-1, 2.5]

$f'(x) = 3x^2 + 6x$

$0 = 3x(x+2)$

$0 = 3x \quad 0 = x+2$

$0 = x \quad -2 = x$

$\boxed{CV: x = \cancel{-2}, 0}$

$\boxed{EV: x = -1, 2.5}$

$f(0) = -4$ MIN

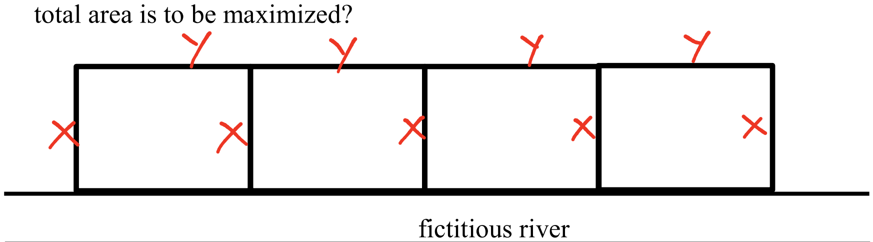
$f(-1) = -2$

$f(2.5) = 30.375$ MAX

Absolute Max: $(2.5, 30.375)$

Absolute Min: $(0, -4)$

#4) A guy whose name is not George wants to make four identical rectangular enclosures along a fictitious river for his fictitious farm, as in the nonfictitious diagram shown below. If he has 2000 inches of fictitious fence (and if the sides along the fictitious river needs no fence of any kind), what should be the dimensions of each enclosure if the total area is to be maximized?



① $A = x \cdot 4y$

$A = x \cdot 4\left(500 - \frac{5}{4}x\right)$

$A = 2000x - 5x^2$

② $P = 5x + 4y$

$2000 = 5x + 4y$

$2000 - 5x = 4y$

$\frac{2000-5x}{4} = y$

$500 - \frac{5}{4}x = y$

③ $A' = 2000 - 10x$

$0 = 2000 - 10x$

$10x = 2000$

$x = 200$

④ $A''(x) = -10$

$A''(200) = $ neg, CCD, MAX

⑤ $500 - \frac{5}{4}(200) = y$

$500 - 250 = y$

$250 = y$

⑥ $A = 4xy$

$= 4(200)(250)$

$A = 20{,}000 \text{ in}^2$

Length of side parallel to river of one enclosure: 250 in

Length of side perpendicular to river of one enclosure: 200 in

$20{,}000 \text{ in}^2$

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#5) An air conditioner manufacturer will be increasing production at the rate of 70 air conditions per week. Revenue from the sale of x air conditioner is $R(x) = 100x - 0.001x^2$ dollars. Find the rate of change of revenue with respect to time when the weekly production level is 550 air conditioners. Give a sentence answer.

x = # of AC
t = time in weeks
R = Revenue

$\frac{dx}{dt} = 70$ AC/week

Find $\left.\frac{dR}{dt}\right|_{x=550}$

$R(x) = 100x - 0.001x^2$

$\frac{dR}{dt} = \frac{d}{dt}(100x) - \frac{d}{dt}(.001x^2)$

$\frac{dR}{dt} = 100\frac{dx}{dt} - .002x\frac{dx}{dt}$

$= 100(70) - .002x(70)$

$\frac{dR}{dt} = 7000 - .14x$

$\left.\frac{dR}{dt}\right|_{x=550} = 7000 - .14(550)$

$= 7000 - 77$

$= \$6923/\text{week}$

Sentence answer: When 550 AC's are sold the revenue is increasing by \$6923 per week.

#6) For $x^2 + 4xy + y^2 = 1$, find $\frac{dy}{dx}$ using implicit differentiation.

$\frac{d}{dx}(x^2) + \frac{d}{dx}(4xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$

$2x + 4\cdot y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$

$\frac{dy}{dx}(4x + 2y) = -2x - 4y$

$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$

$\frac{dy}{dx} = \frac{2(-x - 2y)}{2(2x + y)}$

$\frac{dy}{dx} = \frac{-x - 2y}{2x + y}$

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#7) For $x^{-1} + y^{-1} = 5$, use implicit differentiation to find $\frac{dy}{dx}$ at x = -0.2 and y = 0.1.

$$\frac{d}{dx}(x^{-1}) + \frac{d}{dx}(y^{-1}) = \frac{d}{dx}(5)$$

$$-x^{-2} + -y^{-2}\frac{dy}{dx} = 0$$

$$\frac{-1}{y^2}\frac{dy}{dx} = \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{-y^2}{x^2}$$

$$\left.\frac{dy}{dx}\right|_{(-.2,\,.1)} = \frac{-(.1)^2}{(-.2)^2}$$

$$= \frac{-.01}{.04}$$

$$= -.25$$

$\frac{dy}{dx} =$

At (-0.2, 0.1) $\frac{dy}{dx} =$

#8) A cable television company currently has 10,000 customers and charges $25 per month. A survey by a marketing firm indicated that each decrease of $1 in monthly charges will result in 1000 new subscribers. Determine that monthly charges that will result in a maximum monthly revenue.

x = # of $1 decreases

$p(x) = \$25 - x$

$q(x) = 10{,}000 + 1000x$

$R(x) = p(x) \cdot q(x)$

$= (25 - x)(10{,}000 + 1000x)$

$= 250{,}000 + 25{,}000x - 10{,}000x - 1000x^2$

$R(x) = -1000x^2 + 15{,}000x + 250{,}000$

$R'(x) = -2000x + 15{,}000$

$0 = -2000x + 15{,}000$

$2000x = 15{,}000$

$x = 7.5$

$R''(x) = -2000$

$R''(7.5) =$ neg, CCD, MAX

$q(x) = 10{,}000 + 1000x$

$q(7.5) = 10{,}000 + 1000(7.5)$

$= 10{,}000 + 7{,}500$

$q(7.5) = 17{,}500$

$p = 25 - x$

$p = 25 - 7.5$

$p = 17.5$

$R = p \cdot q$

$R = (\$17.50)(17{,}500)$

$R = \$306{,}250$

Number of $1 decreases: 7.5

Quantity: 17,500

Price: $17.50

Maximum Revenue: $306,250