Basic Integration
Exam Review 4

Integrate each indefinite integral.

#1) $\int(8x^3 - 9x^2 + 10x - 11)dx$

$= 2x^4 - 3x^3 + 5x^2 - 11x + C$

#2) $\int\left(6\sqrt{x} - \frac{1}{x^4}\right)dx$

$= \int(6x^{\frac{1}{2}} - x^{-4})dx$

$= 6\left(\frac{2}{3}\right)x^{3/2} - \left(-\frac{1}{3}\right)x^{-3} + C$

$= 4\sqrt{x^3} + \frac{1}{3x^3} + C$

Castle Greyskull, LLC

#3) As is often the case, George's fantasy gets mixed with reality. George's delusions of grandeur lead him to form an LLC. While filling out his paperwork with the state of Ohio for Castle Greyskull, LLC, George claims the current net worth of Castle Greyskull, LLC is $78 billion and will grow at the rate of $4.4t^{-1/3}$ billion dollars per year t years from now. Find a formula for the net worth of Castle Greyskull, LLC after t years.

$(0, 78)$ t = years

N = \$ billions

$N = \int 4.4t^{-1/3}dt$

$= 4.4\left(\frac{3}{2}\right)t^{2/3} + C$

$N = 6.6\sqrt[3]{t^2} + C$

$78 = 6.6\sqrt[3]{(0)^2} + C$

$78 = 0 + C$

$78 = C$

$N = 6.6\sqrt[3]{t^2} + 78$

Use your formula to find the net worth after 8 years.

$N(8) = 6.6(\sqrt[3]{8})^2 + 78$

$N(8) = 6.6(2)^2 + 78$

$= 6.6(4) + 78$

$= 26.4 + 78$

$= 104.4$ billion

$N(8) = \$104,400,000,000$

Basic Integration
Exam Review 4

Integrate each indefinite integral.

#4) $\int \frac{5}{2x} dx$

$= \frac{5}{2} \ln|x| + C$

#5) $\int e^{3x} dx$

$= \frac{1}{3} e^{3x} + C$

CRAP

#6) Trying to raise money to develop more antidote, George forms the Carcassitis Radiation Association Program where he is both the president and the only employee. He tells prospective investors that CRAP predicts the annual consumption of antidotes will be $0.23e^{0.01t}$ million metric tons per year, where t is the number of years since 2015. Find a formula for the total antidote consumption within t years of 2015

$(0, 0)$

$\boxed{\begin{array}{l} t = \text{years since 2015} \\ A = \text{total antidote consumption millions tons} \end{array}}$

$A = \int 0.23e^{0.01t} dt$

$A = 0.23(100) e^{0.01t} + C$

$A = 23e^{0.01t} + C$

$0 = 23e^{0.01(0)} + C$

$0 = 23e^{0} + C$

$0 = 23 + C$

$-23 = C$

$A = 23e^{0.01t} - 23$

Estimate when the known world reserves of 7 million metric tons will be exhausted.

$7 = 23e^{0.01t} - 23$

$30 = 23e^{0.01t}$

$\frac{30}{23} = e^{0.01t}$

$\ln \frac{30}{23} = \ln e^{0.01t}$

$\ln \frac{30}{23} = 0.01t$

$100 \ln \frac{30}{23} = t$

$26.6 \approx t$

The world reserves will be exhausted during the year 2041.

Basic Integration
Exam Review 4

Use the graphing calculator program Riemann to approximate the area using 10, 100, and 1000 rectangles. Round to two decimal places.

#7) $f(x) = 5x + 3$, a = 2, b = 20

Using 10: $963\ un^2$

Using 100: $1035.9\ un^2$

Using 1000: $1043.19\ un^2$

Use your graphing calculator to find the area under the curve. Use *fnInt* or $\int f(x)dx$

#8) $f(x) = 25 - x^2$, [-5, 5]

$A \approx 166.\bar{6}\ un^2$

#9) Approximate the area under $f(x) = x^2$ from 1 to 2 by three rectangles. Use rectangles with equal bases and with heights equal to the height of the curve at the left-hand edge of the rectangles.

$$\boxed{\Delta x = \frac{b-a}{n} = \frac{2-1}{3} = \frac{1}{3}}$$

$$A = \int_1^2 x^2\,dx$$

$$\approx \Delta x f(x_1) + \Delta x \cdot f(x_2) + \Delta x \cdot f(x_3)$$

$$\approx \frac{1}{3} f(1) + \frac{1}{3} \cdot f\left(\frac{4}{3}\right) + \frac{1}{3} f\left(\frac{5}{3}\right)$$

$$\approx \frac{1}{3}(1)^2 + \frac{1}{3}\left(\frac{4}{3}\right)^2 + \frac{1}{3}\left(\frac{5}{3}\right)^2$$

$$\approx \frac{1}{3}(1) + \frac{1}{3}\left(\frac{16}{9}\right) + \frac{1}{3}\left(\frac{25}{9}\right)$$

$$\approx \frac{9}{27} + \frac{16}{27} + \frac{25}{27}$$

$$A \approx \frac{50}{27}\ un^2$$

Basic Integration
Exam Review 4

#10) Use the Fundamental Theorem of Integral Calculus to find the area under the curve between the given values.

$f(x) = 4 - x^2$, [-1, 1]

$$A = \int_{-1}^{1} (4 - x^2)\,dx$$
$$= \left(4x - \tfrac{1}{3}x^3\right)\Big|_{-1}^{1}$$
$$= \left[4(1) - \tfrac{1}{3}(1)^3\right] - \left[4(-1) - \tfrac{1}{3}(-1)^3\right]$$
$$= \left[4 - \tfrac{1}{3}(1)\right] - \left[-4 - \tfrac{1}{3}(-1)\right]$$
$$= \left[4 - \tfrac{1}{3}\right] - \left[-4 + \tfrac{1}{3}\right]$$
$$= 4 + 4 - \tfrac{1}{3} - \tfrac{1}{3}$$
$$= 8 - \tfrac{2}{3}$$
$$= \tfrac{24}{3} - \tfrac{2}{3}$$
$$A = \tfrac{22}{3}\ \text{un}^2$$

#11) Simplify, then integrate using the Fundamental Theorem of Integral Calculus to find the area under the curve between the given values.

$$\int_1^3 \frac{(t+3)^2}{t^2}\,dt$$

$$= \int_1^3 \frac{t^2 + 6t + 9}{t^2}\,dt$$
$$= \int_1^3 (1 + 6t^{-1} + 9t^{-2})\,dt$$
$$= \left[t + 6\ln|t| - 9t^{-1}\right]\Big|_1^3$$
$$= \left[(3) + 6\ln|3| - \tfrac{9}{3}\right] - \left[(1) + 6\ln|1| - \tfrac{9}{1}\right]$$
$$= \left[\cancel{3} + 6\ln(3) \cancel{- 3}\right] - \left[1 + 6(0) - 9\right]$$
$$= \left[6\ln 3\right] - \left[-8\right]$$
$$= (8 + 6\ln 3)\ \text{un}^2$$

Cotton Swabs Sales

#12) George is starting a new company called *The Slightly Used Company*. George estimates that cotton swabs would sell at a rate of $\frac{7561}{t}$ tips per month during the 2015 calendar year. Find the total number of tips sold from t = 1 to t = 5.

$$\text{Total tips} = \int_1^5 \frac{7561}{t}\,dt$$
$$= 7561 \cdot \ln|t|\Big|_1^5$$
$$= \left[7561 \ln|5|\right] - \left[7561 \cdot \ln|1|\right]$$
$$= \left[7561 \cdot \ln 5\right] - \left[7561(0)\right]$$
$$= 7561 \cdot \ln 5$$
$$\approx 12{,}169 \text{ tips}$$

The total number of tips sold from month 1 to 5 is 12,169

Basic Integration
Exam Review 4

Anti-Derivatives Trigonometric Functions

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + C$$

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \cot(x)\,dx = \ln|\sin(x)| + C$$

More Anti-Derivatives

$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$

$$\int sec\,(x) tan\,(x) dx = sec\,(x) + C$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int csc^{\,2}(x)\,dx = -cot\,(x) + C$$

#13) $\int \sqrt{1-\cos^2(x)}\,dx$

$= \int \sqrt{\sin^2(x)}\,dx$

$= \int \sin(x)\,dx$

$= -\cos(x) + C$

#14) $\int [\sec^2(x) - \csc^2(x)]\,dx$

$= \tan(x) - (-\cot(x)) + C$

$= \tan(x) + \cot(x) + C$

#15) $\int [4\sin^2(x) + 3\tan^2(x) + 4\cos^2(x) - 3\sec^2(x)]\,dx$

$= \int [4\sin^2(x) + 4\cos^2(x) + 3\tan^2(x) - 3\sec^2(x)]\,dx$

$= \int [4(\sin^2(x) + \cos^2(x)) + 3(\tan^2(x) - \sec^2(x))]\,dx$

$= \int [4(1) + 3(-1)]\,dx$

$= \int [4 - 3]\,dx$

$= \int [1]\,dx$

$= x + C$

Basic Integration
Exam Review 4

#16) Find the average value of $f(x) = \frac{2}{3}\sqrt{x}$ from x = 0 to x = 9.

$$Av = \frac{1}{9-0}\int_0^9 \frac{2}{3}x^{\frac{1}{2}}\,dx$$

$$= \frac{1}{9}\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)x^{\frac{3}{2}}\Big|_0^9$$

$$= \frac{4}{81}x^{\frac{3}{2}}\Big|_0^9$$

$$= \frac{4}{81}\left(\sqrt{9}\right)^3 - \frac{4}{81}\left(\sqrt{0}\right)^3$$

$$= \frac{4}{81}(3)^3 - 0$$

$$= \frac{4}{81}(27)$$

$$Av = \frac{4}{3}$$

#17) Find the average value of $f(x) = x^2$ from x = 0 to x = 2.

$$Av = \frac{1}{2-0}\int_0^2 x^2\,dx$$

$$= \frac{1}{2}\left(\frac{1}{3}\right)x^3\Big|_0^2$$

$$= \frac{1}{6}x^3\Big|_0^2$$

$$= \frac{1}{6}(2)^3 - \frac{1}{6}(0)^3$$

$$= \frac{8}{6} - 0$$

$$Av = \frac{4}{3}$$

George's Feet

#18) The temperature of George's cold, sweaty feet at time *x* hours is $T(x) = -x^2 + 2x + 50$ for $0 \le x \le 8$. Find the average temperature between time 2 and 7.

$$AT = \frac{1}{7-2}\int_2^7 (-x^2 + 2x + 50)\,dx$$

$$= \frac{1}{5}\left[-\frac{1}{3}x^3 + x^2 + 50x\right]\Big|_2^7$$

$$= \frac{1}{5}\left[\left[-\frac{1}{3}(7)^3 + (7)^2 + 50(7)\right] - \left[-\frac{1}{3}(2)^3 + (2)^2 + 50(2)\right]\right]$$

$$= \frac{1}{5}\left[\left[-\frac{1}{3}(343) + 49 + 350\right] - \left[-\frac{1}{3}(8) + 4 + 100\right]\right]$$

$$= \frac{1}{5}\left[\frac{-343}{3} + 399 + \frac{8}{3} - 104\right]$$

$$= \frac{1}{5}\left[\frac{-335}{3} + 295\right]$$

$$= \frac{-335}{15} + 59$$

$$\approx 36.7$$

The average temperature of George's feet from 2 to 7 hours is 36.7°.

Basic Integration
Exam Review 4

#19) Find the area between curves that do not cross.

area between	$y=e^x$ $y=\frac{1}{x}$	domain	$1 \le x \le 2$

upper / Lower

$1 \in [1,2]$

$y=e^x$, $y=\frac{1}{x}$

$y=e^{(1)}$, $y=\frac{1}{1}$

$y=e$, $y=1$

upper   Lower

$$A=\int_1^2 \left[e^x-\frac{1}{x}\right]dx$$

$$=\left[e^x-\ln|x|\right]\Big|_1^2$$

$$=\left[e^{(2)}-\ln|2|\right]-\left[e^{(1)}-\ln|1|\right]$$

$$=e^2-\ln 2-e+0$$

$$=\left(e^2-e-\ln 2\right)un^2$$

$$\simeq 3.978\,un^2$$

#20) Find the area bounded by the curves

area between	$y=x^2$ $y=1$

① cross? Yes $(-1,1)$

$x^2=1$

$x^2-1=0$

$(x-1)(x+1)=0$

$x-1=0$, $x+1=0$

$x=1$, $x=-1$

upper / Lower

$0 \in (-1,1)$

$y=x^2$, $y=1$

$y=(0)^2$, $y=1$

$y=0$

Lower   upper

$$A=\int_{-1}^{1}\left[1-x^2\right]dx$$

$$=\left[x-\frac{1}{3}x^3\right]\Big|_{-1}^{1}$$

$$=\left[(1)-\frac{1}{3}(1)^3\right]-\left[(-1)-\frac{1}{3}(-1)^3\right]$$

$$=\left[1-\frac{1}{3}(1)\right]-\left[-1-\frac{1}{3}(-1)\right]$$

$$=\left[1-\frac{1}{3}\right]-\left[-1+\frac{1}{3}\right]$$

$$=1-\frac{1}{3}+1-\frac{1}{3}$$

$$=2-\frac{2}{3}$$

$$=\frac{6}{3}-\frac{2}{3}$$

$$A=\frac{4}{3}\,un^2$$

Basic Integration
Exam Review 4

Integrate by substitution.

#21) $\int (x^2+4)^3 x\,dx$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$= \int u^3 \cancel{x}\left(\frac{du}{2\cancel{x}}\right)$

$= \frac{1}{2}\int u^3\,du$

$= \frac{1}{8}u^4 + C$

$= \frac{1}{8}(x^2+4)^4 + C$

#22) $\int \sqrt{x^3-3x}\,(x^2-1)dx$

$u = x^3-3x$

$\frac{du}{dx} = 3x^2-3$

$du = (3x^2-3)dx$

$\frac{du}{3(x^2-1)} = dx$

$= \int u^{\frac{1}{2}} \cancel{(x^2-1)}\frac{du}{3\cancel{(x^2-1)}}$

$= \frac{1}{3}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{3}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C$

$= \frac{2}{9}\sqrt{(x^3-3x)^3} + C$

#23) $\int_4^9 e^{\sqrt{x}} x^{\frac{-1}{2}} dx$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$= \int_{x=4}^{x=9} e^u \frac{1}{\cancel{\sqrt{x}}}\left(2\cancel{\sqrt{x}}\,du\right)$

$= \frac{1}{2}\int_{x=4}^{x=9} e^u\,du$

$= \frac{1}{2}e^u \Big|_{x=4}^{x=9}$

$= \frac{1}{2}e^{\sqrt{x}} \Big|_4^9$

$= \frac{1}{2}e^{\sqrt{9}} - \frac{1}{2}e^{\sqrt{4}}$

$= \frac{1}{2}e^3 - \frac{1}{2}e^2$

Integrating Trig Functions Using Substitution.

#24) $\int \cos(x)\, e^{\sin(x)}\, dx$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$\frac{du}{\cos(x)} = dx$

$= \int \cancel{\cos(x)} e^u \left(\frac{du}{\cancel{\cos(x)}}\right)$

$= \int e^u\,du$

$= e^u + C$

$= e^{\sin(x)} + C$

#25) $\int \sec(4x)\,dx$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{du}{4} = dx$

$= \int \sec(u)\left(\frac{du}{4}\right)$

$= \frac{1}{4}\int \sec(u)\,du$

$= \frac{1}{4}\ln|\sec(u)+\tan(u)| + C$

$= \frac{1}{4}\ln|\sec(4x)+\tan(4x)| + C$

#26) $\int 3x^2 \sec(x^3)\tan(x^3)\,dx$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$= \int \cancel{3x^2} \sec(u)\tan(u)\left(\frac{du}{\cancel{3x^2}}\right)$

$= \int \sec(u)\tan(u)\,du$

$= \sec(u) + C$

$= \sec(x^3) + C$